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Professor A. S. EDDINGTON, F.R.S., PRESIDENT, in the Chair.

Reginald Wheeler Dobson, M.C., 2 Gerald Road, Oxton, Birkenhead;
Peter Doig, 96 St. George's Square, London, S.W. 1;
Vaman Mukund Gaitonde, M.A., Professor of Mathematics, Guzerat College of Science, Ahmedabad, India;
Franklin Garside, 14 Lime Grove, Hoole, Chester;
Thomas Herbert Le Sbirel Hony, Lloyds Bank House, Fowey, Cornwall;
Col. E. M. Jack, C.M.G., D.S.O., R.E., War Office, Whitehall, London, S.W. 1;
W. M. Lindley, M.A., 26 Park Road, Rugby;
Dr. Knut Lundmark, Upsala, Sweden;
C. E. Kenneth Mees, D.Sc., Director of the Research Laboratory of the Eastman Kodak Co., Rochester, N.Y., U.S.A.;
William Robert Ottway, The Niche, Penton Hook, Laleham, Staines; and
Countess Maria Vicenta de Rivas, Vernon Court, Buckingham Gate, London, S. W. 1,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

Ruby Élise Colomb, Lecturer in Mathematics, Birmingham University (proposed by G. N. Watson);
Ralph Howard Fowler, M.A., Fellow of Trinity College, Cambridge (proposed by E. A. Milne);

Captain H. Lloyd Jenkins, R.N.R., Senior Surveyor, Board of Trade, Newport, Mon. (proposed by E. J. Sharpe);
D. Vaughan Johnston, M.A., Principal, Lewis' School, Pengam, via Cardiff (proposed by W. H. Rees);
Earle Garfield Linsley, Professor of Geography and Astronomy in Mills College, Oakland, California (proposed by C. Burckhalter);
Thomas A. Lumsden, M.A., B.Sc., Assistant Lecturer in Mathematics, Birmingham University (proposed by G. N. Watson);
Christopher Thomas Preece, Lecturer in Mathematics, Birmingham University (proposed by G. N. Watson);
Dr. Frank E. Ross, Research Laboratory, Eastman Kodak Company, Rochester, N.Y., U.S.A. (proposed by L. Silberstein);
Huish Webber, The Vicarage, Studham, near Dunstable, Beds (proposed by Gertrude Bacon); and
Ernest Wunderlich, St. Omer, Waverley, N.S.W., Australia (proposed by W. F. Gale).

Fifty-seven presents were announced as having been received since the last meeting, including, amongst others:—

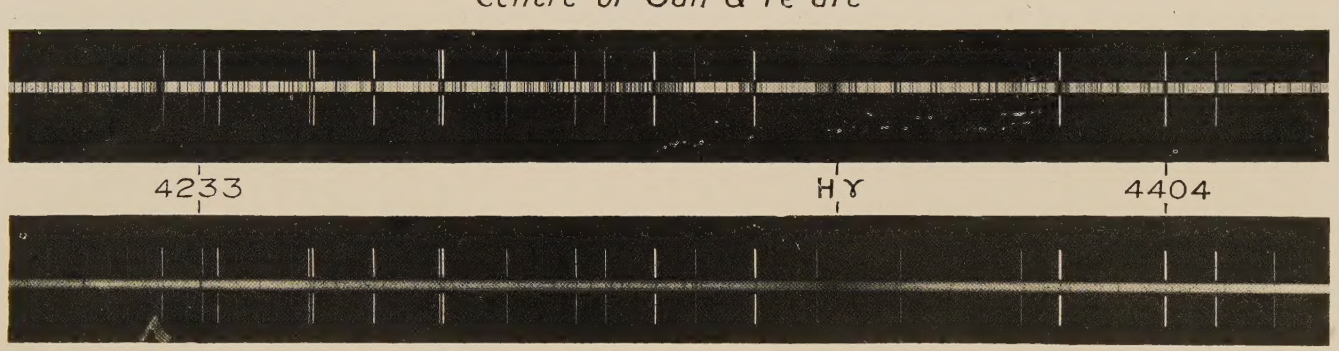
Sydney Observatory: Catalogue of 1068 "Intermediate" Stars.
Mount Wilson Observatory: Photographic Map of the Sun-spot Spectrum, and two transparencies (presented by Professor Hale).

Widened Lines in the Spectrum of Sirius. By J. Evershed. (Plate 10.)

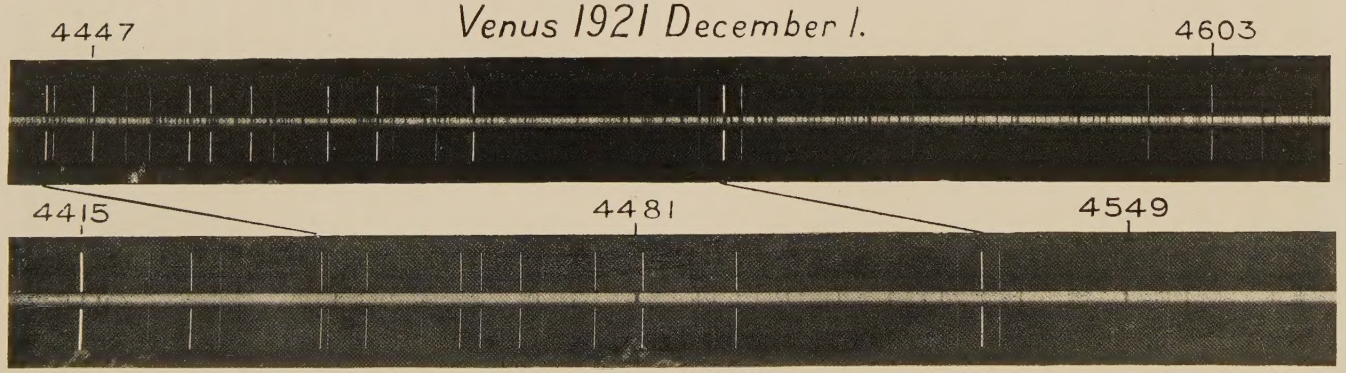
Photographs of the spectrum of Sirius have been obtained at Kodaikánal with the new auto-collimating prism spectrograph specially built for obtaining high-dispersion Venus spectra. The Sirius spectra reveal a peculiarity which has not, so far as I know, been noticed before. The spectrum, as is well known, consists of very broad hazy hydrogen lines and sharply defined metallic lines, of which the majority are the enhanced lines of the elements. In Lockyer's classification Sirius is on the descending side of the temperature curve, and he states that for stars of this class "the hydrogen lines are relatively broad, and the metallic lines thin."*

Our spectra have a dispersion about five times greater than that employed by Lockyer, the scale ranging from about 1·3 A. per mm. to 2 A. per mm., and they include the spectral region between 4220 and 4700. The hydrogen line Hγ shows a quite definite nucleus and a broad shading extending about 10 A. on either side. But the most noteworthy feature in the spectra is the width of the metallic lines, and the uniformity of this character throughout the range of spectrum photo-

* Memoir on the general spectra of certain type stars.



Sirius 1922 February 22.



Sirius 1922 March 25.

graphed. The lines are rather well defined at their edges, and many of them give the impression of being superposed on a broader bright line, but this may possibly be an effect of contrast. If we place side by side with the Sirius spectrum a plate of the Venus or solar spectrum, taken with the same apparatus and the same slit-width, we note the very much greater width and less intensity of the Sirian lines compared with the solar lines. Measurements show that while the finer single lines in the solar spectrum in this region are from 0·05 A. to 0·10 A. in width, nearly all of the Sirius lines, whether weak or strong lines, are between 0·3 A. and 0·4 A. in width. Again, while the majority of the solar lines are intensely dark, showing in fact no photographic action at all, the Sirius lines for the most part are not black but grey, the intensity within them being perhaps one-third or one-half of that of the continuous spectrum.

There are two notable exceptions in the region photographed, the enhanced line of magnesium at 4481·3 and the line at 4549·8: these appear nearly black and cut the spectrum in two at these points. Were it not for the exceedingly narrow lines of the comparison spectrum of the iron arc photographed simultaneously, it might be thought that the star spectrum had been photographed with a very wide slit. The measures give an average width of 0·37 A. for a normally exposed plate, and the variation from line to line is quite small, but the line 4481 is wider, about 0·5 A., with ill-defined edges. The nucleus of the hydrogen line γ is not very much wider, being about 0·6 A., but, owing to the intensity of the shading, the plate is, of course, underexposed here, and the line probably appears wider than it would if projected on a bright background. (Compare the Sirius and solar lines in Plate 10.)

Several hypotheses may be advanced to explain the widening. The broad shading of the hydrogen lines might suggest that there was considerable pressure in the star's atmosphere, and that this may be responsible for the widening of the metallic lines. But it is easy to show that this cannot be. Measures have been made of the extreme widths of the hydrogen lines β, γ, δ, and ϵ in a low-dispersion spectrum of Sirius taken with two 45° prisms and a 5-foot camera. In this the dispersion varies from 18 A. per mm. at H_β to 8 A. per mm. at K, and it is found that the increased dispersion at the violet end just compensates for the apparent increase of width of the more refrangible lines, so that expressed in angstrom units all the four lines are approximately the same width, which is about 20 A.; but the intensity of the continuous spectrum in this plate is greater near H_β where the spectrum is slightly overexposed, and decreases largely towards the violet end where it is underexposed. This no doubt tends to erroneous estimates of the relative widths, and were the spectrum uniformly exposed all along, the width of the bands would probably be found to increase towards the red end, as is also the case with the much narrower hydrogen lines in the Sun.

The fact that the widths do not increase considerably towards the violet shows that it is not a pressure effect, for, according to the experiments of Rossi, the broadening of the hydrogen lines under pressure varies as the inverse third or fourth power of the wave-length, and, even

at a pressure of half an atmosphere only, the line δ was found to be 1·8 times wider than the line β.*

The wings of the hydrogen lines are in all probability produced in the deeper parts of the star's reversing layer, where the density of the hydrogen gas is comparatively great, and if there is no appreciable pressure effect here it cannot occur in the region of iron absorption.

Again, the effect of pressure exceeding an atmosphere on the iron lines is to produce hazy edges and to broaden some lines more than others; but the iron lines in Sirius are all about equally broadened and with defined edges.

The effect of a magnetic field in widening lines by reason of the Zeeman effect may also be ruled out for the same reason: the apparent widening due to the splitting up of the lines into several components would vary greatly from line to line.

It appears that the widening can most probably be explained as a Doppler effect, but this may be due to several causes. If Sirius were a spectroscopic double star there would be a periodical change in the width of the lines, if not an actual doubling, at times of greatest relative velocity in the line of sight. Now in a plate taken on 1922 February 21, the iron arc lines at 4308, 4425·9, and 4404·9 appear to show signs of resolution into two, especially 4404; but none of the other iron lines show this, and it may be due to some accidental alignment of the silver grains. The double star hypothesis may be tested by comparing photographs taken on different dates. We have available plates taken on 1922 January 4, February 18, February 21, March 14, and March 25, and in addition two underexposed grating spectra, one obtained on 1917 February 4 and the other on 1920 March 17. Comparing all these plates, there appears to be no sensible variation in the width of the lines, and only on the plate of 1922 February 21 is there a slight indication of doubling. It is not probable therefore that the light is derived from two stars in relative motion.

According to the kinetic theory of gases the velocity of the luminous atoms will be proportional to the square root of the absolute temperature. This movement causes a Doppler widening of spectrum lines, which for homogeneous radiations will be proportional to $\sqrt{\frac{T}{M}}$, T being the absolute temperature and M the atomic weight or mass. Taking the absolute temperature of the Sun's atmosphere at 6000°, and that of Sirius at 10,000°, we find that the Sirius lines will be widened in the ratio of 1·3 only compared with the solar lines; also that the hydrogen lines should be $7\frac{1}{2}$ times wider than the iron lines. Actually the measures show that the Sirius lines are nearly 4 times wider than the corresponding solar lines, and the nucleus of the hydrogen line γ is only about 1·6 times the width of the neighbouring iron lines; whilst the wings of the hydrogen lines are over 50 times the width of the iron

* R. Rossi, *Astrophysical Journal*, 34, 301, 302. The significance of these experiments with regard to pressure in solar or stellar atmospheres does not appear to have been realised. They show that pressures exceeding half an atmosphere do not exist in the reversing layers of Sun or any stars showing distinct hydrogen lines.

lines. Evidently the high temperature of Sirius will not account for the widened lines.

Another hypothesis which may be suggested is a rapid rotation of the star. If Sirius is rotating on an axis lying nearly normal to the line of sight, we might expect that the integrated effect of the line-of-sight velocities over the disc would widen the lines. In the case of the Sun, we find that the lines in general sunlight when photographed under very high dispersion have practically the same widths as lines in light from a limited region of the Sun, as the centre of the disc: the only difference is a slight haziness at the edges, which is seen on comparing close double lines. The relative velocity at the equatorial limbs of 3·9 km./sec. has apparently no effect in widening the lines. If, however, we block out the centre of the Sun, and integrate the light from a narrow annulus, the lines are widened on the average about one and a third times. The actual increase of width for the region about 4427 is, according to my measures, about 0·034 A., equivalent to 2·3 km./sec. Thus, even in this case, we do not get the full widening due to the equatorial rotation velocity.

General sunlight differs only very slightly from light taken from the centre of the Sun's disc because of the great falling off in intensity near the limb, so that in a properly exposed plate only the light from the central part of the disc is effective. But it would appear that in Sirius the density of the reversing layer is probably so low that atomic scattering of the photospheric light does not reduce the intensity near the limb of the star to the same extent. The line-of-sight velocities near the east and west limbs may therefore be much more effective in widening the lines than in the case of our Sun. There is no doubt that the relative intensity of the ultra-violet part of the spectrum in stars of class A compared with those of class G indicates the absence of the so-called smoke-layer (atomic scattering) in the former class.

The actual measured widths of the Sirius lines would imply a linear speed of over 12 km./sec. at each limb. But, judging by the experiment with the light from an annulus of the Sun, this must be largely underestimated, and there is the probability that the axis is not exactly normal to the line of sight. Recent estimates of the diameter of Sirius indicate a value about 1·9 times that of the Sun, so that taking 12 km./sec. as representing the minimum value of the equatorial velocity, the angular speed will be over three times that of the Sun, a complete rotation taking eight days or less. This high speed of rotation seems improbable considering that Sirius is in an earlier stage of evolution, and is therefore less condensed than the Sun: one would expect too that the lines would be more diffused at the edges than they actually are.

The only remaining hypothesis I can suggest is that the Sirian atmosphere is subject to much more violent convection than is the solar atmosphere. If there are radial convection currents with velocities of over 12 km./sec. rising and falling, the lines in the light from the central part of the star disc would be widened to the amount shown.

An analogy can be found in the Sun, but on a very much smaller scale. A high-dispersion photograph of the spectrum when the slit lies across a well-defined image of the Sun reveals innumerable small dis-

placements of the lines equivalent to velocities of the order of a few tenths of a kilometre per second. These irregularities tend to widen the lines in general sunlight, but to an extent probably not exceeding 0·008 A.

Light on the question could be obtained by photographing high-dispersion spectra of other bright stars of the same class to ascertain whether the widening of the lines is peculiar to Sirius or is a feature common to stars of type A.

Kodaikánal:
1922 *April* 12.

Note on the Pressure in the Reversing Layer in Stars.

By R. d'E. Atkinson.

(*Communicated by F. A. Lindemann.*)

The stars of classes K5–M are well known to be distinguished by a spectrum showing signs (sometimes very marked) of absorption by TiO_2.* The effects of temperature and pressure on the equilibrium of this compound are therefore of considerable interest; if they are known, such knowledge of the temperatures in these stars as we already possess will allow of the calculation of the pressure in the reversing layer. The temperature and pressure interact, of course, on any equilibrium (unless the reaction occurs without change of volume) in such a manner that knowledge of either allows us to infer from the presence or absence of the compound a limit for the other. The present paper is an attempt to apply this principle to the calculation of the pressure in the reversing layer from the equilibrium calculated by Nernst's heat theorem. The evidence of the pressure shift † has been held to show that the pressure is between 1 and 10 atmospheres, but the arguments brought forward here seem to require a value of about a millionth of this.

The present theory of giant stars already implicitly assumes that matter at very low densities ($5 . 10^{-5}$ or less) can still emit a continuous spectrum. For since it is probable that the masses of all stars are nearly equal, their densities must vary with $S^{-\frac{3}{2}}$ if S is the surface area; it is assumed that S varies directly as the total radiation, and since this may be 1000 times as great for a giant as for a dwarf star (whose density is about 1), the density of the giant star may be 1/30000 of that of the dwarf. The presumption is that the region in which the continuous spectrum is produced is of the same order of density in dwarfs of the same temperature. Were this not so, it would be difficult to account for the similarity of the spectra.

The fact that a continuous spectrum can be emitted by matter at such a low density may be accounted for in various ways; it might, for

* Fowler, *P.R.S.*, 79, 509 (1907).
† Fabry and Buisson, *C.R.*, 148, 688 (1909), etc.

instance, be due to the Stark effect alone. The mean molecular field in a gas where ions are present to a total mass per c.c. of 10^{-6} gr. is, according to Debye,* about 10^4 volts per cm. Since it is a chaotic field it will not appear to split up lines, but to broaden them; the hydrogen lines would be broadened by this strength of field to about 4 Å, and therefore the average line should be broadened perhaps to one-tenth of this, in which case a line-spectrum containing $2\frac{1}{2}$ lines per Ångstrom would become continuous. The only difference in appearance (apart from the size) of giant and dwarf stars at the same temperature would then be due to the difference in the thickness (and therefore the total mass) of the reversing layer. Presumably the reversing layer is more or less in convective equilibrium; that is to say, the temperature and pressure gradients are determined by the ordinary formula for adiabatic expansion, so that a mass of gas that rushes up to a region of lower pressure is cooled according to the equation $pv^\gamma = \text{constant}$: we then have

$$\frac{d\mathrm{T}}{dh} = \frac{\mathrm{M}g}{\mathrm{R}} \frac{\gamma - 1}{\gamma} \quad \text{and} \quad \frac{dp}{dh} = \frac{\mathrm{M}g}{\mathrm{R}} \frac{p}{\mathrm{T}},$$

where M is the mean molecular weight, γ the mean ratio of the specific heats, g the value of gravity, and R the gas constant. If the zero, *i.e.* the density at the photosphere, is the same for giant and dwarf stars, the only quantity that will depend on the star is g; this varies inversely as the square of the radius, so that the reversing layer in a giant star will be very much deeper than in a dwarf. The above equations are not, of course, strictly true, because the equilibrium becomes at great heights rather radiative than convective; were this not so, there would be a region near the top of the layer where the temperature was so low that however low the pressure any compound might exist. Actually, of course, the chromosphere is still hot enough to radiate considerably, so that the expression for $\frac{d\mathrm{T}}{dh}$ needs an important correction term.†

It is not easy to calculate what differences in appearance between giants and dwarfs at the same temperature should be expected as a result of this; roughly, it is clear that with the present assumptions no substantial difference is to be expected, and no great difference has apparently yet been detected.

Thus there seems to be no *a priori* reason to suspect the thermochemical argument if it leads to very low pressures; in fact, high pressures would probably be more difficult to reconcile with other considerations.

The effect of pressure on an equilibrium is, of course, a direct consequence of the law of mass-action. The equilibrium of TiO_2 is determined by the quantity $\mathrm{K}_p \equiv \frac{p_{\mathrm{Ti}} p_{\mathrm{O}_2}}{p_{\mathrm{TiO}_2}}$, where p denotes the partial pressure of each substance; K_p depends only on the temperature and the units in which pressure is measured. On inserting values the effect of a change of pressure is at once apparent; if $\mathrm{K}_p = 10^{-4}$, we could equally

* *Phys. Zeitschr.*, **20**, 160 (1919).
† See Lindemann, *P.M.*, **38**, 680 (1919).

well have $p_{Ti} = p_{O_2} = 10^{-3}$, $p_{TiO_2} = 10^{-2}$ total $1{\cdot}2 \cdot 10^{-2}$, or $p_{Ti} = p_{O_2} = 10^{-5}$, $p_{TiO_2} = 10^{-6}$, giving a total $\frac{1}{570}$ of the former; this drop of pressure clearly causes such dissociation that p_{TiO_2} drops from $83\frac{1}{2}$ per cent. to 4·8 per cent. of the corresponding total. Owing to this very sudden falling off of the percentage of a compound formed with decrease of volume, when the pressure passes a certain point, no accurate measure of the percentage present is needed to determine the pressure: if the compound is there in appreciable quantities at all the pressure is more than a certain amount, and conversely.

The temperature difference between the photosphere and the upper limit at which an absorbing substance exists can be obtained reasonably well. For, putting E_ν for the "black body" emissivity at frequency ν, h for Planck's constant $6{\cdot}55 \cdot 10^{-27}$, k for Boltzmann's constant $1{\cdot}346 \cdot 10^{-16}$ and c the velocity of light, since $e^{\frac{h\nu}{kT}}$ is large compared with 1,

$$E_\nu = \frac{8\pi h}{c^3} \cdot \nu^3 e^{-\frac{h\nu}{kT}}, \quad \therefore \quad \frac{\partial E_\nu}{E_\nu} = \frac{h\nu}{k} \cdot \frac{\partial T}{T^2}, \quad \text{whence} \quad \log \frac{E_\nu}{E_\nu'} = \frac{h\nu}{k} \cdot \frac{T_1 - T_2}{T_1 T_2};$$

putting $T_1 = 3500°$, $T_2 = 3300°$, and $\nu = 5 \cdot 10^{14}$, we get $E_\nu = 1{\cdot}5\, E_\nu'$. It is not certain that the intensities of lines are (even in a star) equal to E_ν for that temperature and frequency, but it is fairly probable, and (more especially important for the following inquiry) almost certain in the case of an extended band-spectrum. We may then take the TiO_2 as being perhaps 200° colder than the photosphere. Since the temperature of this is about 3000° to 3500° in the stars under consideration,* we have now a very fair means of estimating the pressure in these stars if we can obtain the variation of K_p with temperature.

In the first place, it is clear that the pressures will be so low that we must be dealing with a pure gas-reaction: the vapours cannot possibly be in equilibrium with solid or liquid metal or oxide. For, since $Mp = RT\rho$ where M is the mean molecular weight, p the total pressure and ρ the density, then if, as above, $\rho = 10^{-5}$ (which is, of course, an upper limit), giving M the rough value 50,

$$p = \frac{8{\cdot}31 \cdot 10^7 \cdot 3500 \cdot 10^{-5}}{50}$$

$$= 5{\cdot}8 \cdot 10^4 \text{ dynes cm.}^{-2} = \tfrac{1}{18} \text{ atm. approximately.}$$

The partial pressures of Ti and TiO_2 cannot form a very large fraction of this, and one is probably justified in putting them at less than 10^{-2} atms. In this case, therefore, all that is required is the equilibrium of the gas-reaction.

For a gas-reaction occurring with evolution of heat Q_{290} at 290° abs., and between gases whose specific heats and chemical constants are denoted by C_p and C respectively, we have the equations

$$Q_{290} - Q_0 = \int^{290} \Sigma C_p dT$$

* Russell, *Nature*, 93, 228 (1914).

and

$$-A = RT ln K_p = -Q_0 + T\int\frac{dT}{T^2}\int \Sigma C_p dT + 2{\cdot}3RT\Sigma C$$

(writing $ln = 2{\cdot}3 \log$),

where in the summations Σ the substances disappearing in the reaction are counted as positive, and Q_0 is the heat which would be developed by the reaction at 0° abs. Of course, Q_{290} (as between the gases) cannot be measured; what is measured is $Q'_{290} = Q_{290} - \lambda_{Ti} + \lambda_{TiO_2}$, where λ is the latent heat necessary to vaporise the substance at 290°. The values of λ could be accurately obtained if the melting and boiling points of the metal and oxide, and the latent heat of each transformation at the normal temperature, and the specific heats of the solids, liquids, and gases at all temperatures were all known; the latent heats at transition temperatures could also be deduced fairly closely (by modifications of Trouton's rule) if the temperatures of melting and boiling (more particularly boiling) were known. Almost all these values are lacking. The melting points, however, are known and are not far apart; if the boiling points also are not far apart, the latent heats of vaporisation (per gram molecule) will probably not be very far from equal; in any case, it is their difference only that is wanted, and this should not be very great. On the whole, since practically the only available item of evidence is the approximate equality of the melting points, it is probably as accurate as can be looked for (besides being much the simplest) to assume that the latent heats cancel out, and to take the observed heat of reaction, 215600 kilo-cal., as that which would be developed by a pure gas-reaction at the same temperature (290°). It is true that for a substance boiling at 3500, if we adopt the formula $\lambda = 8{\cdot}5T \log T$, we get a latent heat of 10^5 cal., and a consideration of specific heats shows that at room temperature this would not be much decreased; but even if we admit that the difference of the latent heats might amount to 10^4 cal., it will be found that this does not affect the result very seriously, and almost certainly the error involved will not change the order of magnitude. Another source of uncertainty is in the values to be given to the specific heats of the gases. The following values have been adopted, and again they are probably near enough to give a tolerably correct result. For Ti, $C_p = 5{\cdot}0$; for O_2, as a mean of the results of Nernst* and Pier,† $C_p = 6{\cdot}6 + 6{\cdot}3 \,.\, 10^{-4} T$; and for TiO_2, by comparison with Nernst's values for SO_2 and CO_2, modified by the fact that the molecule of TiO_2 may be supposed to be a good deal more firmly bound than either of these, $C_p = 7{\cdot}5 + 2{\cdot}5 \,.\, 10^{-3} T$. It must be admitted that the formula for O_2 makes the value of C_p rise, above 3000°, higher than the theoretical maximum (9) for a diatomic gas, but the difference is over a small enough range, and at a high enough temperature, to be neglected.

The chemical constants are comparatively unimportant. That for O_2 is given by Nernst as 2·8; that for Ti (which is presumably monatomic) is ·92, since it is given by the formula $C = -1{\cdot}608 + \frac{3}{2} \log M$ (where M is the molecular weight); and that for TiO_2 may be taken (by comparison with CO_2 and SO_2) as 3·3.

* *Zeit. Electroch.*, **17**, 272 (1911). † *Ibid.*, **16**, 897 (1910).

Inserting the figures in the above equations, we get

$$Q_0 = 215{\cdot}6 \,.\, 10^3 - \int^{290} [5 + 6{\cdot}6 - 7{\cdot}5 + (0{\cdot}6 - 2{\cdot}5)10^{-3}T]dT = 214{\cdot}6 \,.\, 10^3$$

and

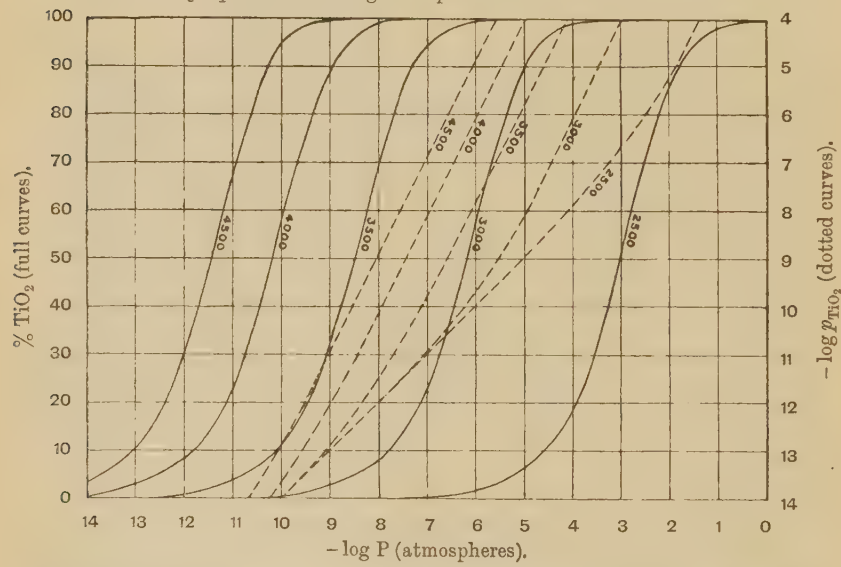
$$-A = RT ln K_p = -214{\cdot}6 \,.\, 10^3 + 9{\cdot}45T \log T - 9{\cdot}5 \,.\, 10^{-4}T^2 + 1{\cdot}92T,$$

and putting in values for T we obtain the following table :

T	2500	3000	3500	4000	4500
$10^{-3}A$	135·5	118·8	102·6	85·9	70·1
$-\frac{A}{2{\cdot}3RT} = \log K_p$	−11·9	−8·7	−6·41	−4·70	−3·42

From these values of $K_p \equiv \frac{p_{Ti} \cdot p_{O_2}}{p_{TiO_2}}$ the actual pressures can be obtained by assuming $p_{Ti} = p_{O_2}$. In the diagram the continuous curves

TiO_2 Equilibrium at High Temperatures and Low Pressures.



show the percentage (pressure) of the mixture which is not dissociated, *i.e.* the TiO_2, while the broken curves give the actual pressure of the TiO_2, both plotted against the "combined pressure" P of all the reacting substances (*i.e.* $P = p_{Ti} + p_{O_2} + p_{TiO_2}$). Thus, for instance, if the pressure P is 10^{-6} atms. and the temperature 3500, 30 per cent. of the mixture $Ti + O_2 + TiO_2$ will be TiO_2; while if the temperature were 4000 the fraction would be only $1\frac{1}{2}$ per cent. Again, if the pressure P were 10^{-9} atms., 10 per cent. of the mixture would be oxide at 3000 and practically none at 3300; (the dotted curve shows $p_{TiO_2} = 4 \,.\, 10^{-12}$). If we accept Professor Russell's value 3100°–3200° for the temperature of

the continuous spectrum, 3000° should be about that of the absorbing layer, and it appears unlikely therefore that the "combined pressure" can be much below 10^{-9}. Again, Professor Russell gives 4200° as the temperature of the continuous spectrum in class Ko where no trace of TiO_2 is found. The temperature of the reversing layer may be taken as 4000° in these stars, and the TiO_2 spectrum should certainly appear if P were as large as 10^{-5}, and probably if P were over 10^{-6}. The general shape of the curves indicates, indeed, that P is probably between 10^{-7} and 10^{-8}. For, according to the figures for the temperature quoted above, the change in appearance due to a drop in temperature from 4200° to 3200° is not greater than the change from 3200° to 3100°. The TiO_2 may be taken as 200° cooler than these in each case.

Titanium is the tenth commonest element in the earth's crust. Unless it is assumed that it forms an inordinately small proportion of the whole in the reversing layer of stars, the absence of the oxide in the hotter stars implies that the pressure in the reversing layer is much lower than is commonly supposed. If it is assumed that P is as little as 10^{-3} of the total pressure, for instance, this total can scarcely exceed 10^{-2} atms., while 10^{-5} or 10^{-6} would seem more probable, and the limits may be placed with fair confidence, as far as this investigation goes, at 10^{-7} and 10^{-3} respectively.

The combined pressure has been given on the assumption that the oxygen and titanium are present in equivalent proportions: this may well not be true, since there is a great excess of O_2 over Ti on the earth. At the same time, it is for practical purposes only another aspect of the fact that the proportion of the combined pressure to the total pressure is not known. It makes little change whether the difference between them is due to more oxygen or to neutral elements.

The line of argument adopted is, of course, applicable whenever a suitable compound can be found. In particular, it should be possible to determine the pressure in sun-spots from observations on magnesium oxide if that is present; titanium oxide is found, and the temperature is probably rather higher than the K5 star photosphere, indicating a higher pressure; if the magnesium oxide spectrum is confirmed, there will be further data to work on.

It may be noted that if the effect of temperature on the minimum pressure consistent with a continuous spectrum were large at the temperatures in question, the disappearance of TiO_2 with increase of temperature could probably be explained by this alone (*i.e.* even if K_p were constant), since it might be the case that at higher temperatures we could not see into the star to a great enough depth; the spectrum would be continuous before the pressure was high enough to allow the compound to exist. The same effect, of course, but to a greater extent, may be expected when K_p varies. Though it may be possible, therefore, that TiO_2 does exist even in hotter stars than observation indicates, it would do so at such a depth as to be masked by the continuous spectrum of higher layers. It is only observed in stars which are so cool that in the reversing layer the pressure is high enough to prevent dissociation at that temperature.

To account for the occurrence of the TiO_2 spectrum is thus seen to

be fairly simple; two other questions suggest themselves, namely, why do no other compounds appear until much lower temperatures, and why does this particular spectrum become such a dominant factor in some of the stars? The first question is susceptible of definite treatment, while the second is still almost entirely a matter of conjecture. There are few oxides formed with so high a heat of reaction as TiO_2, except such as have considerably larger numbers of atoms in the molecule, such as Al_2O_3. There are a number with heats of reaction of about 142,000 calories, and only two or three atoms to the molecule, but the difference between this value and the 216,000 calories for TiO_2 is very marked. An exception to the above statement should, however, be made in favour of CeO_2 and ThO_2, both of which (and more especially the latter) have higher heats of formation. In fact, there seems every reason for supposing that ThO_2 should be visible, even at much higher temperatures than TiO_2, and since even a comparatively small quantity should give rise to an absorption spectrum, its absence, if it is absent, must be due to some other cause, presumably its rarity or possibly its high molecular weight. Several halides, especially those of the alkaline earth metals, give high heats of reaction, and this question is receiving further consideration. As one might, of course, expect, some endothermic compounds are stable at these temperatures, and, in point of fact, some cyanogen compounds are observed.

The problem of the extraordinary relative strength of the TiO_2 spectrum in some cases is another matter. All that seems possible at present is to point out that, once compounds start forming in appreciable proportions, the ordinary conditions of convective equilibrium are materially altered. Whether dissociation as a result of the drop in pressure, or association as a result of the drop in temperature, would be the prevailing feature in an upcurrent would require some calculation. It is certainly of interest (*e.g.* for the theory of sun-spots), since in one case the compound will be acting as an explosive and in the other as a buffer. Again, it is not improbable that the somewhat marked tendency to variability observed in stars in whose atmosphere compounds are found may be partially explained along these lines. However that may be, the conditions occurring during chemical action may well modify the atomic changes which give rise to the line-spectra.

In conclusion, I wish to express my gratitude to Professor Lindemann, at whose instance this investigation was undertaken, for the advice and help which he has given at every stage in its progress.

Clarendon Laboratory, Oxford:
1922 *May* 12.

Observations of Nova Persei (1901), 1921–1922.

By W. H. Steavenson.

In a recent paper (*M.N.*, 82, 49) brief reference was made to Nova Persei, which was still under observation at the time of writing. In the present note are given details of the observations made between 1921 August 6 and 1922 March 25. During the first two months of this period most of the observations were made with the 6-inch Wray refractor. Unfortunately, a change of residence made it necessary to dismantle this instrument early in October, and it was not possible to re-erect it until the beginning of May. Consequently, there have been rather serious breaks in the continuity of the work, quite apart from intervals of bad weather. The few observations obtained since October are due to the kindness of various friends who have granted the use of their instruments.

The observations show very clearly that the Nova is now an irregular variable; which has, of course, been known for some time. The light-curve, so far as it has been possible to draw it, is exceedingly complex. It has the appearance of being produced by the constant interaction of many periodicities, both long and short. Professor Turner pointed out, at the November meeting, that the mean magnitude of the Nova fell steadily from 1914 to a minimum in 1918, and has since been as steadily rising. Assuming this change to be truly periodic, it points to a very long period, one of at least six years. In addition to this, Professor Turner has found, by harmonic analysis of Barnard's observations, a shorter period of 396 days (*M.N.*, 81, 426). These results rest not so much on individual observations as on mean values for the magnitude during certain periods. The observations have usually been made at intervals of at least several days (sufficiently frequent to bring out a long period), but no attempt appears to have been made to study the light-curve in great detail. Consequently the minor oscillations have been ignored; which seems a pity, as they are undoubtedly real, and may well lead to the ultimate explanation of the whole curve. At present it is not clear whether each of these oscillations has a short period of its own or whether all share one and the same long period. In the latter case we should expect them to repeat themselves in the same order at intervals, and there is some slight ground for believing that they do so about every 114 days. This point can only be settled by a series of very frequent (preferably nightly) observations extending over several months. In our climate this is an impossible task for one observer. On the other hand, a combination of the results of several workers is not likely to give a satisfactory curve, since the personal equations of the observers would be liable to mask the small oscillations, whose amplitude is of the order of $0^m{\cdot}3$. A single observer, working in a climate such as that of Egypt, would stand the best chance of securing a conclusive series of observations.

The estimates which follow are probably correct to within $0^m{\cdot}1$. Those indicated as being approximate may be, perhaps, as much as $0^m{\cdot}25$ in error. The Nova is difficult to observe owing to the nebulosity in which it is involved. This nebulosity is not visible with direct vision

in apertures below 12 inches, but it is evident with averted vision in a clear sky, and has the effect of making the Nova appear brighter than it actually is.

The comparison stars are those used by Barnard, who published their magnitudes (determined by Seares) in *M.N.*, **80**, 711.

The instruments used are referred to as under:—

G = 28-inch Grubb refractor of the Royal Observatory.
P = 12¼-inch Calver reflector, belonging to Rev. T. E. R. Phillips.
O = 12-inch reflector, made by and belonging to Dr. L. W. Oliver.
A = 9-inch reflector, made by and belonging to Commander M. A. Ainslie.
M = 6-inch Cooke refractor, belonging to Mr. G. Merton.
Ga = 6-inch Grubb refractor, belonging to Major G. E. Galbraith.
S = 6-inch Wray refractor, belonging to the writer.

Following are the individual estimates of magnitude.

Date.	Mag.	Instrument.
1921 Aug. 6	12·32	S.
8	12·42	S.
27	13·18	S.
29	13·36	S.
30	13·11	S.
Sept. 3	12·96	S.
5	12·86	S.
7	12·74	S.
8	12·70	P.
10	12·74	S.
15	12·48	S.
16	12·48±	S.
21	12·64±	S.
22	12·74±	S.
24	12·95	S.
25	12·74	G.
	*12·64	S.
27	12·69	S.
28	12·74	S.
29	12·69	S.
30	12·74	S.
Oct. 1	12·59	S.
2	12·49	S.
4	12·51	S.
5	12·48	S.
10	12·32	O.
11	12·27	A.
1921 Oct. 15	12·48	M.
16	12·37	M.
17	12·27±	M.
23	12·32	M.
24	12·32	M.
Nov. 27	12·90	M.
28	13·03	M.
29	13·02	M.
Dec. 20	13·30±	A.
31	12·36	A.
1922 Jan. 16	12·64	A.
17	12·48±	A.
20	12·48	A.
30	12·64±	A.
31	12·64	A.
Feb. 1	12·64	A.
18	12·74	A.
20	12·79	A.
21	12·87	A.
24	13·23	Ga.
26	13·52	Ga.
Mar. 4	13·17	M.
7	13·08±	A.
20	<13·2	A.
21	<13·2	A.
25	<13·2	A.

* The difference of $0^{m}\cdot1$, though small, probably represents a real change, as the observations were made more than two hours apart, and the magnitude of the Nova was rising rapidly at the time.

Observations of (*Ch.* 7085) *RT Cygni in* 1917–22.

By A. N. Brown, M.A.

I have observed this variable as frequently as possible since 1917, and the 277 observations given below are in continuation of those published in *Monthly Notices*, **77**, 550–555. Light-curves which have been drawn from them show 10 maxima and 9 minima, as follows:—

Maxima.				Minima.			
No.	Date.	Julian Date.	Ded. Mag.	No.	Date.	Julian Date.	Ded. Mag.
18	1917 May 6	1355	7·2	19	1917 Sept. 5	1477	11·2
19	1917 Dec. 6	1569	8·2	20	1918 Mar. 9	1662	11·4
20	1918 June 15	1760	7·5	21	1918 Sept. 28	1865	11·4
21	1918 Dec. 22	1950	7·3	22	1919 Mar. 28	2046	10·9
22	1919 June 26	2136	6·3	23	1919 Oct. 16	2248	11·0
23	1920 Jan. 15	2339	7·4	24	1920 Apr. 20	2435	11·1±
24	1920 July 9	2515	7·5	25	1920 Oct. 21	2619	11·0
25	1921 Jan. 12	2702	7·3	26	1921 Apr. 28	2808	11·1
26	1921 July 31	2902	7·4	27	1921 Nov. 6	3000	10·9
27	1922 Feb. 11	3097	7·0				

The number given on the left of each date in the above table indicates the number of maxima and minima which have occurred since I first began to observe this star on 1908 Jan. 12.

There are so many peculiarities in the light-curve during the last five years that, although anxious to economise space as much as possible, I think it desirable to add a few notes upon them. All the remarks which follow refer to the maxima. When the light is fainter than about 10·0 mag. it appears to change steadily, and without any marked irregularities.

Notes on the Maxima.

No.

18. A very rapid rise after J.D. 1334, the date of the last published observation, when the estimate was 8·6 mag. The light reaches 7·2 mag. on 1355, then declines to 7·4, and rises to a secondary maximum—again of 7·2 mag.—on 1388. It seems, therefore, somewhat misleading to give *one* date for maximum. Mr. Edward Prentice, who has submitted these observations to harmonic analysis, and has been kind enough to comment upon some of my results, informs me that his calculated date seems to come just midway between the two.

19. This seems to me the most peculiar part of the curves under consideration. The maximum appears to be fairly well determined, but it was preceded by an extremely disturbed state of the light —a rise to 9·5 mag., reached on J.D. 1508, being followed by a fall of half a magnitude. This was succeeded by a rapid rise of 0·6 mag., after which came a pause, until the steady rise to

No.

maximum begins. This is the faintest maximum (8·2 mag.) which I have observed so far, and it was preceded by a slow fall, lasting 122 days, to minimum No. 19. (For similar conditions in the past *cf. M.N.*, **72**, 417 and **77**, 551.)

20. Here a pause preceded the rise, and the light was apparently unsteady during the time of maximum.
21. A pause on the rise, and a still more decided pause on the fall.
22. A pause is followed by a pointed maximum, and there is another pause on the descending branch of the curve.
23. This maximum (7·4 mag.) is apparently followed by a slight but decided fall, and then succeeded by a secondary maximum (7·5 mag.).
24. It seems as if exactly the same thing occurred here.
25. A difficult maximum to determine, as it is very much prolonged. Mr. Prentice suggests that, as there is a gap of 10 days (2702–2712) in the observations, perhaps one observation during these days would have made a difference.
26. A slight pause before the rise.
27. Two pauses on the ascending branch. The second is prolonged, and is followed by a rapid "flare up" to maximum. The fall proceeds at a regular pace.

The mean period derived from the maxima and minima of 1917–22 as given above is 192·0 days.

The observations have been arranged in exactly the same way as those of the previous series, and the same telescopes were used as before, namely, 5-inch and 3-inch refractors, with powers varying from 25 to 200.

The Observations.

Date. 1917	Julian Date.	Comparisons.	Ded. Mag.	Date. 1917	Julian Date.	Comparisons.	Ded. Mag.
Apr. 19	1338	4−1, 5+1	7·7	July 1	1411	5−1, =7	8·0
21	1340	4−0·5, 5+1	7·7	6	1416	10−2, 14+1; 13+2	8·5
25	1344	3−7, 4+2	7·4				
May 1	1350	3−6·5, 4+3	7·3	15	1425	14−4·5, 19+2	9·1
6	1355	3−5, 4+5	7·2	19	1429	15−3, 20+1·5; 18−1, 19+1	9·2
8	1357	3−6·5, 4+3	7·3				
24	1373	3−7, 4+2	7·4	22	1432	19−1·5, 23+1	9·4
28	1377	3−7, 4+2	7·4	25	1435	25−1, 30+2	9·8
June 3	1383	3−6, 4+4	7·3	Aug. 14	1455	=43	10·6
8	1388	3−5, 4+5	7·2	18	1459	43−3, 44+1	10·9
11	1391	3−7, 4+2	7·4	Sept. 2	1474	Not seen. <43	<10·6
13	1393	3−7, 4+2	7·4	8	1480	44−2	11·2
17	1397	3−8, 4+2	7·5	11	1483	44−1	11·1
21	1401	=5; 6+0·5	7·8	12	1484	44−1	11·1
26	1406	=5, 7+1	7·9	14	1486	=44	11·0

The Observations—continued.

Date.	Julian Date.	Comparisons.	Ded. Mag.
1917			
Sept. 20	1492	43 − 3, 44 + 1	10·9
22	1494	43 − 2·5, 44 + 1	10·9
24	1496	= 43	10·6
26	1498	41 − 1·5, 43 + 1	10·5
Oct. 4	1506	22 − 1·5, 25 + 1; 23 + 2	9·7
6	1508	= 22; = 23	9·5
10	1512	23 − 1, 25 + 1	9·6
17	1519	25 − 2, 30 + 1	9·9
23	1525	25 − 2, = 30	10·0
26	1528	22 + 1; 23 + 1	9·4
Nov. 4	1537	16 − 4·5, 19 + 1	9·2
7	1540	16 − 4, 19 + 1	9·2
12	1545	14 − 5, 19 + 1	9·2
14	1547	14 − 2, 19 + 4	8·9
16	1549	14 − 1, 19 + 4	8·8
25	1558	7 − 3, 10 + 1	8·3
Dec. 2	1565	7 − 2, 10 + 1·5	8·2
6	1569	7 − 2, 14 + 4·5; 10 + 2	8·2
11	1574	7 − 2, 10 + 2	8·2
14	1577	7 − 3, 10 + 1	8·3
17	1580	7 − 3, 10 + 1	8·3
25	1588	14 − 1, 19 + 5·5	8·8
27	1590	14 − 1	8·8
1918			
Jan. 3	1597	14 − 4, 19 + 1	9·1
7	1601	14 − 4, 19 + 1	9·1
11	1605	19 + 1; 22 + 3	9·2
30	1624	43 − 1, 44 + 3	10·7
Feb. 8	1633	43 − 3, 44 + 1	10·9
16	1641	44 − 2	11·2
22	1647	44 − 2	11·2
Mar. 1	1654	44 − 4	11·4
14	1667	44 − 4	11·4
Apr. 1	1685	44 − 1	11·1
6	1690	= 44	11·0

Date.	Julian Date.	Comparisons.	Ded. Mag.
1918			
Apr. 12	1696	43 − 2, 44 + 2	10·8
18	1702	41 − 1, 43 + 1	10·5
25	1709	23 − 4; = 30	10·0
27	1711	22 − 2; = 25	9·7
May 2	1716	14 − 5, 19 + 1	9·2
8	1722	14 − 3; 19 + 4	8·9
13	1727	= 14, 19 + 5	8·7
17	1731	= 14	8·7
28	1742	3 − 8, 4 + 1·5	7·5
31	1745	4 − 1, 5 + 0·5	7·7
June 5	1750	3 − 7, 4 + 2·5	7·4
10	1755	= 4, 5 + 1	7·7
13	1758	= 4	7·6
15	1760	= 4	7·6
16	1761	3 − 7·5, 4 + 2; 5 + 5	7·4
22	1767	4 + 1	7·5
25	1770	3 − 7·5, 4 + 2	7·4
29	1774	= 4, 5 + 1	7·7
July 2	1777	4 − 0·5, 6 + 2; 5 + 3	7·6
6	1781	4 − 0·5, 5 + 1; 6 + 3	7·6
8	1783	4 − 1, 5 + 0·5; 6 + 2	7·7
12	1787	4 − 1, = 5	7·8
16	1791	4 − 3; 5 + 1	7·8
18	1793	5 − 1, 7 + 1	7·9
21	1796	5 − 1·5, 7 + 1	7·9
29	1804	14 − 4, 19 + 1	9·1
Aug. 9	1815	25 − 2, 30 + 1	9·9
12	1818	41 + 1	10·3
14	1820	41 + 1	10·3
18	1824	= 41	10·4
20	1826	41 − 1, 43 + 0·5	10·5
24	1830	43 − 1, 44 + 3	10·7
31	1837	= 44	11·0
Sept. 15	1852	Not seen. < 43	< 10·6
23	1860	44 − 3	11·3
28	1865	44 − 4	11·4
Oct. 3	1870	44 − 3	11·3
6	1873	44 − 3	11·3

The Observations—continued.

Date.	Julian Date.	Comparisons.	Ded. Mag.
1918			
Oct. 8	1875	44−3	11·3
13	1880	43−3, 44+0·5	10·9
24	1891	=43	10·6
Nov. 3	1901	30−1, 41+3	10·1
7	1905	25−1·5, 30+2	9·8
9	1907	25−1, 30+2·5	9·8
12	1910	23−1·5, 25+1; 22−2	9·7
15	1913	=22; =23	9·5
29	1927	7−2·5, 10+1	8·3
Dec. 7	1935	5+1; 7+3	7·7
12	1940	4−0·5, 5+1	7·7
17	1945	3−7·5, 4+2	7·4
23	1951	3−6·5, 4+3	7·3
26	1954	3−7, 4+2	7·4
29	1957	4−2; 5+2	7·7
1919			
Jan. 2	1961	5−1·5, 7+1	7·9
5	1964	7−1	8·1
9	1968	7−1, 10+3	8·1
16	1975	7−1·5, 10+3	8·1
22	1981	10−1, 14+2	8·5
Feb. 10	2000	23−2; =25	9·7
Mar. 30	2048	43−3, 44+1	10·9
Apr. 8	2057	43−3, 44+1	10·9
19	2068	41−1, =43	10·6
24	2073	41+1·5	10·3
May 8	2087	14−5; 19+1; 22+2	9·2
12	2091	14−3, 19+3	9·0
17	2096	10−2, 14+1	8·6
19	2098	10−2, 14+1	8·6
22	2101	7−2, 10+2	8·2
28	2107	4−1, 5+1	7·7
30	2109	4−1·5; 5+1	7·7
June 3	2113	3−7, 4+2	7·4
9	2119	3−2, 4+7	6·9
1919			
June 11	2121	3−2, 4+7	6·9
16	2126	3−1	6·8
21	2131	2−1, 3+0·5	6·6
26	2136	2+2	6·3
July 8	2148	2−1, 3+0·5	6·6
15	2155	3−2, 4+7	6·9
17	2157	3−6·5, 4+3	7·3
22	2162	3−8, 4+1·5	7·5
30	2170	3−8·5, 4+1	7·5
Aug. 6	2177	7−3; 10+1	8·3
9	2180	7−3, 10+1	8·3
15	2186	14−1	8·8
21	2192	14−6·5, 22+2	9·3
27	2198	=19	9·3
Sept. 6	2208	30−2; 41+1	10·3
11	2213	41−1, =43	10·6
19	2221	43−2, 44+1	10·9
28	2230	44+1	10·9
Oct. 2	2234	=44	11·0
12	2244	44−1	11·1
15	2247	=44	11·0
17	2249	=44	11·0
26	2258	=44	11·0
31	2263	44+1	10·9
Nov. 11	2274	43−2, 44+1	10·9
19	2282	25−6, 41+1	10·3
21	2284	25−2; 30+1	9·9
26	2289	=19, 22+1; 23+2	9·3
Dec. 2	2295	14−4, 19+1	9·1
4	2297	14−4·5, 19+2	9·1
8	2301	14−1	8·8
19	2312	7−1, 10+3	8·1
31	2324	4−1, 5+1	7·7
1920			
Jan. 5	2329	4+0·5	7·6
8	2332	3−8, 4+1·5	7·5

The Observations—continued.

Date.	Julian Date.	Comparisons.	Ded. Mag.
1920			
Jan. 16	2340	3 − 7·5, 4 + 2	7·4
19	2343	3 − 8, 4 + 1·5	7·5
24	2348	= 4	7·6
31	2355	4 − 1, 5 + 1	7·7
Feb. 8	2363	4 + 1·5	7·5
11	2366	3 − 8, 4 + 1·5	7·5
Mar. 6	2390	14 − 4·5, 19 + 2	9·1
18	2402	22 − 1 ; 23 − 1 ; 25 + 1	9·6
Apr. 21	2436	44 − 1	11·1
May 7	2452	41 − 1, = 43	10·6
13	2458	30 − 3, 41 + 1	10·3
19	2464	25 − 1, 30 + 2	9·8
21	2466	23 + 1 ; 25 + 1	9·5
23	2468	19 + 1	9·2
29	2474	14 − 4·5, 19 + 2	9·1
June 1	2477	14 − 3, 19 + 3	9·0
6	2482	10 − 1, 14 + 1	8·5
8	2484	10 − 1, 14 + 1·5	8·5
18	2494	4 − 1, 5 + 0·5	7·7
July 14	2520	3 − 8, 4 + 1·5	7·5
18	2524	3 − 8·5, 4 + 1	7·5
24	2530	4 − 0·5, 5 + 1	7·7
Aug. 6	2543	3 − 7·5, 4 + 2 ; 5 + 3	7·5
9	2546	4 + 1 ; 5 + 3	7·5
15	2552	4 − 0·5, 5 + 1	7·7
20	2557	4 − 1, = 5	7·8
22	2559	5 − 1, 7 + 1	7·9
26	2563	7 − 3, 10 + 1	8·3
Sept. 7	2575	14 − 5·5, 19 + 1	9·2
18	2586	= 25	9·7
Oct. 2	2600	43 − 1, 44 + 2·5	10·7
6	2604	43 − 2, 44 + 2	10·8
10	2608	43 − 2·5, 44 + 1	10·9
19	2617	= 44	11·0
24	2622	44 + 1	10·9
30	2628	43 − 3, 44 + 1	10·9
Nov. 1	2630	44 + 1·5	10·9
4	2633	43 − 3, 44 + 1	10·9
11	2640	= 43	10·6
20	2649	25 − 1, 30 + 2	9·8
23	2652	= 25	9·7
Dec. 1	2660	14 − 5 ; 19 + 1	9·2
9	2668	14 − 6, 22 + 2 ; 19 + 2	9·2
21	2680	7 − 1, 10 + 2·5	8·1
28	2687	3 − 6·5, 4 + 3	7·3
1921			
Jan. 7	2697	3 − 6·5, 4 + 3	7·3
12	2702	3 − 6·5, 4 + 3	7·3
22	2712	3 − 6·5, 4 + 3	7·3
31	2721	3 − 7, 4 + 2	7·4
Feb. 2	2723	3 − 7, 4 + 2	7·4
17	2738	3 − 7, 4 + 2	7·4
26	2747	10 − 1, 14 + 2	8·5
Mar. 2	2751	14 − 0·5	8·7
14	2763	22 + 1	9·4
24	2773	41 + 2	10·2
30	2779	43 − 3, 44 + 1	10·9
Apr. 3	2783	43 − 1, 44 + 3	10·7
7	2787	43 − 3, 44 + 1	10·9
12	2792	44 + 1	10·9
28	2808	44 − 1	11·1
May 8	2818	44 − 1	11·1
15	2825	= 44	11·0
19	2829	43 − 2·5, 44 + 1	10·9
21	2831	43 − 2, 44 + 2	10·8
June 6	2847	25 − 2, 30 + 1	9·9
11	2852	= 25	9·7
23	2864	14 − 1, 19 + 5	8·8
28	2869	14 + 1	8·6
July 5	2876	5 − 0·5, 7 + 1 ; 10 + 5	7·9
8	2879	5 + 1	7·7

The Observations—continued.

Date.	Julian Date.	Comparisons.	Ded. Mag.
1921			
July 12	2883	4 − 1, 5 + 0·5	7·7
20	2891	3 − 7·5, 4 + 2	7·4
25	2896	3 − 7, 4 + 2	7·4
Aug. 2	2904	3 − 7, 4 + 2	7·4
8	2910	3 − 7, 4 + 2	7·4
25	2927	4 − 1 ; 5 + 1	7·7
Sept. 3	2936	7 + 0·5	8·0
7	2940	7 − 3, 10 + 1	8·3
21	2954	14 − 4, 19 + 1	9·1
27	2960	19 + 1 ; 22 + 2	9·3
29	2962	22 − 1 ; = 23	9·6
Oct. 4	2967	30 − 2, 41 + 2	10·2
8	2971	41 + 1	10·3
14	2977	43 − 1, 44 + 2·5	10·7
16	2979	43 − 2·5, 44 + 1	10·9
23	2986	44 + 1	10·9
Nov. 6	3000	44 + 1	10·9
10	3004	44 + 1	10·9
23	3017	43 − 3, 44 + 1	10·9
26	3020	43 − 3, 44 + 1	10·9
Dec. 4	3028	41 − 1, 43 + 0·5	10·5
20	3044	= 19 ; 22 + 2	9·3
22	3046	14 − 5, 19 + 1	9·2
28	3052	14 − 4, 19 + 2	9·1
31	3055	14 − 1	8·8
1922			
Jan. 3	3058	11 − 2, 14 + 1	8·6
10	3065	4 + 1 ; 5 + 2	7·6
16	3071	3 − 7, 4 + 2	7·4
Feb. 1	3087	3 − 7, 4 + 2	7·4
9	3095	3 − 6, 4 + 4	7·3
11	3097	3 − 3, 4 + 6·5	7·0
13	3099	3 − 5, 4 + 5	7·2
18	3104	3 − 7, 4 + 2	7·4
26	3112	4 − 1 ; 5 + 0·5	7·7
Mar. 7	3121	7 − 1·5, 10 + 3	8·1
20	3134	14 − 1	8·8
28	3142	19 + 2 ; 22 + 4	9·1
Apr. 19	3164	43 − 2·5, 44 + 1	10·9

The Comparison Stars Used.

Hagen's Number.	Magnitude. H.P.	Magnitude. Hagen.
2	6·50	
3	6·67	
4	7·64	
5		7·8
6		7·9
7		8·0
10	8·39	
11		8·4
13	8·54	
14	8·67	
15		8·8
16	8·71	
18		9·1
19		9·3
20		9·3
22		9·5
23		9·5
25		9·7
30		10·0
41		10·4
43		10·6
44		11·0

Ludgrove, New Barnet, Herts:
1922 *April* 29.

Notes on Two Long-Period Variables, RT Cygni and U Persei.
By Edward Prentice.

RT Cygni. Ch. 7085. Harvard 194048.

Observations of this star from 1908 to 1917 by Mr. A. N. Brown appear in *Monthly Notices*, 77, No. 7, p. 550, and four earlier numbers, while some dates of maxima, going back to the year 1890, are to be found in *Harvard Annals*, 55; the latter are, unfortunately, not continuous.

The elements given by Chandler in his revised third Catalogue are J.D. 2410514 + 190·5 E. Discovered by Pickering 1890.

Mr. Brown deduced 192·4 days for the period from his observations, 192 days was therefore adopted for the purpose of a harmonic analysis of the light-curve, but it is evident from the drop in the phases of the first harmonic that this period is somewhat too long. These phases, together with the coefficients of the first harmonic and the ranges in magnitude, are shown in Table I.

TABLE I.

E.	ϕ_1.	C_1.	Range Mag.	E.	ϕ_1.	C_1.	Range Mag.
38	207°	2·99	5·6	52	173°	2·68	3·9
39	182	2·88	3·7	53	161	2·75	4·0
40	186	3·11	4·0	54	160	3·10	4·0
41	187	2·79	3·6	55	155	2·89	3·9
42	181	3·08	3·7	56	155	3·06	4·0
43	179	3·09	4·1	57	178	2·52	4·0
44	198	2·76	4·1	58	150	2·53	3·9
45	165	2·86	3·8	59	169	3·07	4·0
46	169	2·94	3·7	60	145	2·82	4·6
47	166	2·99	3·4	61	172	2·99	4·1
48	156	2·60	4·3	62	163	3·05	3·6
49	156	2·78	3·7	63	152	3·20	3·7
50	149	3·09	3·9	64	146	3·23	3·7
51	164	3·26	3·7	65	146	3·15	3·8

The range given for the period E 38–39 is possibly accidental; if not, it is a peculiar feature in the star; it is noticeable too that in the neighbourhood of epochs 41, 49, 56, 65, that is, at intervals of nearly eight periods or roughly 1520 days, there are halts in the phases. Owing to considerable gaps in the Harvard observations it was hardly

practicable to draw satisfactory curves for some of the periods, and therefore a harmonic analysis has been made for only two periods of these observations, with the result that we get for their mean curve the expression

$$3{\cdot}11\cos(\theta-180^\circ)+{\cdot}11\cos(2\theta-110^\circ{\cdot}1)+{\cdot}18\cos(3\theta-45^\circ{\cdot}7),$$

while an analysis of Mr. Brown's observations gives

$$3{\cdot}06\cos(\theta-180^\circ)+{\cdot}14\cos(2\theta-110^\circ{\cdot}5)+{\cdot}04\cos(3\theta-2^\circ{\cdot}7).$$

These two results seem to be satisfactorily in accord, and enable us to place the star in Group II.

TABLE II.

E.	Obs.	Comp.	O−C.	C−U.	E.	Obs.	Comp.	O−C.	C−U.
32	2416605 J.D.				50	20029	20	+ 9	−19·0
33	6792	93	− 1	− 7·5	51	0208	10	− 2	−19·5
34	6981				52	0414	09	+ 5	−11·0
35	7170	67	+ 3	−14·5	53	(0598)	10	−12	− 0·5
36	7360	57	+ 3	−15·0	54	0787	91	− 4	−10·0
37	7552	51	+ 1	−11·5	55	0990	92	− 2	+ 0·5
38	7740	37	+ 3	−16·0	56	1172	81	− 9	− 1·0
39	7927	29	− 2	−14·5	57	1355	70	−15	− 2·5
40	8122	21	+ 1	−13·0	58	1569	68	+ 1	+ 5·0
41	8309	08	+ 1	−16·5	59	1760	62	− 2	+ 8·5
42	8508	08	0	− 7·0	60	1950	47	+ 3	+ 3·0
43	8692	02	−10	− 3·5	61	2136	37	− 1	+ 2·5
44	8870	74	− 4	−22·0	62	2339	46	− 7	+21·0
45	9085	83	+ 2	− 3·5	63	2515	32	−17	+16·5
46	(9270)	58	+12	−19·0	64	2702	07	− 5	+ 1·0
47	9437	44	− 7	−23·5	65	2902	06	− 4	+ 9·5
48	9648	52	− 4	− 6·0	66	3097	88	+ 9	+ 1·0
49	9822	24	− 2	−24·5					

The dates of maximum for epochs 33–66 have been computed from the curve, and in Table II. are compared with those of a uniform period, taking Chandler's elements, that is, a period of 190·5 days, the residuals being set out in column 5. These indicate that the period increased from E 33 to E 65, but on taking some of the earlier epochs from 6–32, and comparing such observed dates as are available with the same uniform period, we have evidence of a decrease during these periods (Table III.). It may be, then, that there is a cyclic change in the lengths of the periods, but at present this can be only conjectural.

TABLE III.

E.	Obs.	190·5 d. Uniform.	O−U.	E.	Obs.	190·5 d. Uniform.	O−U.
6	2411670 J.D.	57	+13·0	20	4324	24	0
8	2048	38	+10·0	21	4532	14·5	+17·5
10	2433	19	+14·0	22	4700	05	− 5·0
12	2806	00	+ 6·0	23	4893	95·5	− 2·5
13	2996	90·5	+ 5·5	25	5273	76·5	− 3·5
14	3180	81	− 1·0	27	5650	57·5	− 7·5
15	3384	71·5	+12·5	28	5850	48	+ 2·0
16	3569	62	+ 7·0	30	6213	29	−16·0
17	3749	52·5	− 3·5	31	6409	19·5	−10·5
18	3941	43	− 2·0	32	6605	10	− 5·0
19	4147	33·5	+13·5				

All the observed dates given in these tables have been taken either from the *Harvard Annals*, or else, in the case of Mr. Brown's observations, are those deduced by him.

If we divide our periods into groups and take the mean for each group, we find that there is a slight fall at first and then a rise in the number of days (Table IV.). The mean of these seven groups is 190·5 days, which is the number given by Chandler for the star's period.

TABLE IV.

E.	Days.	E.	Days.
0–8	191·7	40–48	190·7
8–16	190·1	48–56	190·5
16–32	189·7	56–64	191·2
32–40	189·6		

U Persei. Ch. 678. Harvard 015354.

Chandler's third Catalogue gives the elements J.D. 2411353+318 E with the note "large irregularities." Discovered by Mrs. Fleming 1890. The Harvard elements are J.D. 2410050+320 E. Observed dates of maxima from 1892 onwards appear in the *Harvard Annals*, 55, which dates have been taken for the epochs prior to those covered by Mr. Brown's observations published in *Monthly Notices*, 71, 79, while adopting Chandler's starting-point. As Hagen's values were used by Mr. Brown for his comparison stars, it was thought advisable to reduce his observations to the Harvard values; this alteration increased the range in his mean curve by 0·7 mag. The period adopted for a harmonic analysis of the light-curve was 317·3 days.

From such analysis of the Harvard mean curve for seven periods we get

$$2{\cdot}97\cos(\theta - 180^\circ) + {\cdot}46\cos(2\theta - 31^\circ{\cdot}4) + {\cdot}12\cos(3\theta - 323^\circ{\cdot}2),$$

and from Mr. Brown's observations, including those which have not yet been published, up to the current year

$$2{\cdot}92\cos(\theta - 180^\circ) + {\cdot}47\cos(2\theta - 76^\circ{\cdot}5) + {\cdot}15\cos(3\theta - 0^\circ{\cdot}2),$$

here then the phases of the second and third harmonics indicate Group II.

TABLE I.

E.	ϕ_1.	C_1.	Range Mag.	E.	ϕ_1.	C_1.	Range Mag.
16	201°	2·80	2·8	27	182°	2·76	3·3
17	218	3·00	3·2	28	176	2·55	3·5
18	204	2·85	3·0	29	184	2·75	3·5
19	207	2·09	3·0	30	191	2·69	3·6
20	191	3·12	2·8	31	191	2·53	3·5
21	178	2·80	2·5	32	187	3·11	3·1
22	192	2·34	3·1	33	187	2·49	3·2
23	193	2·54	2·9	34	194	2·22	3·2
24	193	3·15	2·7	35	198	2·43	3·5
25	191	3·18	3·1	36	214	2·65	2·6
26	178	2·84	3·2	37			

In Table I. the phases of the first harmonic show considerable jumps for the periods E 17–18, 20–21, 26–27, 36–37; also between epochs 24 and 32 there is seen to be a steady increase in the range followed by a change in the reverse direction.

The result of a comparison between the computed dates and a uniform period reckoned from J.D. 2411353 + 317·2 E appears in column 5, Table II.

TABLE II.

E.	Obs.	Comp.	O–C.	C–U.	E.	Obs.	Comp.	O–C.	C–U.
17	2416783 J.D. 66		+17	+20·6	28	20194	08	−14	−26·6
18	7060	64	− 4	+ 1·4	29	0523	28	− 5	−23·8
19	7408	98	+10	+18·2	30	0892	95	− 3	+26·0
20	7693	99	− 6	+ 2·0	31	1156	63	− 7	−23·2
21	8010	02	+ 8	−12·2	32	1488	77	+11	−26·4
22	8322	25	− 3	− 6·4	33	1801	98	+ 3	−22·6
23	8627	19	+ 8	−29·6	34	2117	09	+ 8	−28·8
24	8959	52	+ 7	−13·8	35	2425	27	− 2	−28·0
25	9292	97	− 5	+14·0	36	2739	42	− 3	−30·2
26	9571	72	− 1	−28·2	37	3089	87	+ 2	− 2·4
27	9874	67	+ 7	−50·4					

Some information about the earlier periods, for which we have not sufficient observations for plotting curves, may be gathered from Table III.

From these tables it seems that the star's period has been decreasing; but it also looks as if the reverse process is now commencing, and that for the next few periods we may expect an increase. It does not appear in any case to vary in a consistent manner, for we get spasmodic jumps such as those at E 25 and 30.

TABLE III.

E.	Obs.	317·2 d. Uniform.	O−U.	E.	Obs.	317·2 d. Uniform.	O−U.
3	2412332 J.D.	04·6	+27·4	9	4236	07·8	+28·2
5	2952	39	+13·0	11	4857	42·2	+14·8
	3271	56·2	+14·8	12	5173	59·4	+13·6
7	3581	73·4	+ 7·6	14	5820	93·8	+26·2
8	3895	90·6	+ 4·4	16	6420	28·2	− 8·2

In Table IV. the periods are taken in groups; here for the mean of each 16 periods, or roughly 5070 days, we get practically the same result, though the mean for 37 periods is 317·2 days.

TABLE IV.

E.	Days.		E.	Days.	
0–8	317·8	316·7 d.	16–24	317·4	316·8 d.
8–16	315·6		24–32	316·1	

In the *Astro. Journal*, 16, 202, with reference to observed dates of maxima, allusion is made to a light-curve plotted from observations of this star 1895–96, in which there are three crests with two decided depressions between them, such as would account for discordances in dates of maxima as given by different observers; similar crests appear in curves derived from the subsequent observations.

The method adopted with both these stars for computing the dates of maximum has been to take a segment from the upper part of the curve and to find the $\bar{x}$ co-ordinate of this segment.

Mr. Brown has kindly supplied his unpublished observations of these stars for the analysis, the continuity of his observations adding much to their value for the purpose.

Ashtead:
1922 *May* 5.

Two Variable Stars in Gemini. By A. Stanley Williams.

The variability of the following stars was noticed recently when comparing photographs taken with a 4·4-inch portrait lens. The places are very rough.

(1) R.A. = $7^h\ 07^m\ 40^s$. Decl. = $+26°\ 15'$ (1855).

Photographic Observations.

1900	April	19.	Mag. = $10\frac{1}{2}$
	,,	25.	,, = $10\frac{1}{2}$
1901	,,	13.	,, = $10\frac{1}{2}$
1902	,,	6.	,, = 12
	,,	27.	Not visible, below 12 mag.
1903	,,	22.	,, ,,

The star was invisible in a $6\frac{1}{2}$-inch reflector from 1921 December 22 to 1922 February 18, when it was 12 mag., and from thence it rose to $10\frac{1}{2}$ mag. on 1922 April 26.

(2) R.A. = $7^h\ 16^m\ 45^s$. Decl. = $+33°\ 38'$ (1855).

Photographic Observations.

1900	April	19.	Mag. = 11
	,,	25.	,, = 11
1901	,,	13.	,, = $11\frac{1}{2}$
1902	,,	6.	Not visible, below 12 mag.
	,,	27.	,, ,,
1903	,,	22.	,, ,,

The star was invisible in a $6\frac{1}{2}$-inch reflector from 1921 December 22 to 1922 January 21, when it was 12 mag., and from thence it rose to $9\frac{1}{2}$ mag. on 1922 April 26.

The periods of both these stars are evidently long.

Polvarth, St. Mawes :
1922 *April* 27.

A probably Variable Star in Gemini. By A. Stanley Williams.

Six photograhs taken with a 4·4-inch portrait lens indicate slight but apparently certain variability in a star of about 10 mag. in R.A. = $7^h\ 12^m\ 52^s$, Decl. = $+31°\ 07'$ (1855). Recent visual observations seem to confirm this, but the variation so far observed is not conspicuous, whilst the star is difficult to observe. Attention is only now called to the probable variability of the star because of it having been used by Dr. Graff as a comparison star in his observations of the short-period

Cepheid variable RR Geminorum,* and its use may have affected his conclusions. Attention has lately been directed by Professor Turner † to the possible variability of a comparison star used for the observation of a variable star of a similar type to RR Geminorum, and its influence on the results.

Polvarth, St. Mawes:
1922 *April* 16.

On the Tawny Hue of Jupiter's Equatorial Zone.

By A. Stanley Williams.

Recent observations show that the tawny ‡ hue of the equatorial zone has now practically disappeared. It may, therefore, be useful to derive from my observations the date of maximum tawny coloration. The observations may be summarised as under, it being premised that the few notes in the years immediately preceding 1918 refer to the zone as being white and bright.

1918 August and September. Zone not white, but had a slight ochrey tinge.

1920 January to May. The tawny hue was fairly deep and conspicuous, whilst the zone appeared markedly duller than the other bright parts of the planet.

1920 October to December. The tawny hue, very evident in October, appeared to be less so in December, but the dullness of the zone seemed more pronounced.

1921 May. Tawny hue not at all conspicuous, and the dullness of the zone less evident.

1921 December to 1922 April. Tawny hue practically all gone, and the zone no longer dull.

The maximum tawny coloration from my observations occurred about the middle of 1920, but the exact date is not easily fixed.§ A better determination should be possible when the observations of other observers are available, particularly those by the members of the Jupiter Section of the British Astronomical Association.

Polvarth, St. Mawes:
1922 *April* 27.

* *Mitteilungen der Hamburger Sternwarte*, No. 8, p. 62. It is the star lettered *b* by Graff, whose observations likewise indicate variability, the difference between *b* and his star *a* varying to the extent of 0·6 mag., and that between *b* and *c* to the extent of 0·5 mag.

† *M.N.*, **81**, 74.

‡ *M.N.*, **80**, 467.

§ The computed date is 1920·7. This makes the eighth occasion on which the tawny hue has now been observed, omitting the doubtful one of 1767.

Recent Jupiter Observations. By Frank Sargent.

In May 1921 a large dark "spot" was observed at the north edge of the South Equatorial Belt on the planet Jupiter.

In a note to the Society I gave some particulars respecting it, referring especially to the abnormal rotation period, such being almost unique.*

Remembering that about forty years had elapsed since any similar "period" had been recorded, it may not be without significance that in less than a year from the apparition of 1921 another spot has appeared very similar in its distinctive features to its recent predecessor.

Unfortunately the "spot" of 1921 appeared so late in the "opposition" that an extended series of observations was not possible. The present object, appearing earlier, has yielded a more useful series.

The new "spot" is situated at the north edge of the South Equatorial Belt. It may, perhaps, with even greater accuracy be described as at the south edge of the Equatorial Zone, as the object appears to be detached from the South Equatorial Belt.

It was first observed from here during the evening of March 31, when its longitude according to Sys. I. was found to be 172°·9. Since that date it has retrograded more than thirty-six degrees, having been observed at about longitude 209° on May 2. Adopting these positions, a mean period for the intervening seventy-eight rotations is 9 hrs. 51 mins. 15·4 secs., forty-five seconds in excess of the Sys. I rate, or about one minute longer than "periods" derived for similar Jovian latitudes during recent years.

A closer examination of the observations seems to point to another interesting feature. It appears highly probable that the "period" has lengthened during the time covered by them.

The observations are as follows:—

1922 March	31	On C.M.	9 hrs.	27 mins.	Long. 172·9 Sys. I.
April	6	,,	13	11	,, 177·6
,,	8	,,	14	27	,, 180
,,	9	,,	10	10	,, 181·4
,,	11	,,	11	24·5	,, 182·9
,,	16	,,	9	42	,, 190·4
,,	18	,,	10	58	,, 193
,,	23	,,	10	15	,, 199·8
,,	24	,,	14	48	,, 200·8
,,	29	,,	13	4	,, 207·2
May	2	,,	11	4	,, 209·5

Plotting these observations, a lengthening of the rotation period will be at once apparent, unless the first observation is "adjusted" to an extent that is an altogether improbable one.

During the first eight or ten days a value of 9 hrs. 51 mins. 6 secs.

* See *Mon. Not.*, 81, 8, p. 521.

is indicated, whilst the remaining observations require a period of 9 hrs. 51 mins. 18·8 secs.

On March 10 the region where the spot would have been had it then existed was under observation. Nothing was recorded concerning it; whereas on March 31 the object was without doubt the most obvious feature on the planet, the similarity with the "spot" of 1921 May being at once recognised and recorded.

On April 11 a similar but fainter "spot" was seen preceding the principal one by about five degrees, and has since shared the latter's abnormal movement. By the 27th the order of intensity had changed. At the time of writing both have lost much of their intensity, and seem likely soon to pass beyond telescopic reach.

In view of the close resemblance in appearance and peculiarity of motion between the "spot" of 1921 and the more recent one, a similarity in method of origin is highly probable, and, if mechanically this is of a volcanic order, then in point of origin also.

Unfortunately, our knowledge of Jovian rotation is limited to that of an inconstant surface; but should these spots be the precursors of many such, then their early discovery coupled with a useful series of observations of them may in time enable us to penetrate by legitimate inference to somewhat lower depths.

The University Observatory, Durham:
1922 *May* 10.

On the Application of Lambert's Theorem to the Determination of Orbits. By M. T. Subbotin.

(*Communicated by the Secretaries.*)

§ 1. The utilisation of the well-known Lambert's theorem for computing the orbit of a body, moving round the sun according to Kepler's laws, has been the subject of much investigation. But the only methods of practical interest are those proposed for orbits with very large semi-axis a.

A new form of Lambert's theorem permits us to extend considerably the use of this fundamental relation.*

This new form is

$$(1) \quad \frac{1}{4a} = \frac{\tau}{r+r'} - \frac{\varepsilon^2}{4\theta^2}, \quad \theta = k(t'-t),$$

where r and r' are the radii-vectores of the body at the times t and t' respectively; s is the chord, joining its positions at these times.

* The substance of §§ 1 and 5 was given in a note sent to *Comptes Rendus* in November 1921. The use of the new form of Lambert's relation in the method of variation of geocentric distances (§ 6) was the subject of a note, which was sent to the Royal Astronomical Society in October 1921. But in the present Russian conditions I do not know if these notes have reached their destination.

If we put

$$R=\frac{r+r'}{4a},\quad c=\frac{s}{r+r'},$$

we shall have, for the case of elliptic motion,

$$(2)\quad \begin{cases} \tau = R+\dfrac{16R^3c^2}{(\epsilon-\sin\epsilon-\delta+\sin\delta)^2}, \\ \sin\dfrac{\epsilon}{2}=\sqrt{R(1+c)},\quad \sin\dfrac{\delta}{2}=\sqrt{R(1-c)}; \end{cases}$$

for hyperbolic motion the corresponding formulæ being

$$\begin{cases} \tau = R-\dfrac{16R^3c^2}{(\sinh\epsilon-\epsilon-\sinh\delta+\delta)^2} \\ \sinh\dfrac{\epsilon}{2}=\sqrt{-R(1+c)},\quad \sinh\dfrac{\delta}{2}=\sqrt{-R(1-c)}; \end{cases}$$

and, for parabolic motion, we shall have

$$\tau=\left[\frac{3c}{(1+c)^{\frac{3}{2}}-(1-c)^{\frac{3}{2}}}\right]^2.$$

As in all cases in practice τ differs very little from unity and changes slowly, the equation (1) is very convenient for finding a from $r+r'$, s, and $t'-t$. It is only necessary to have some suitable method for calculating τ for every pair of values of arguments R and c, upon which this quantity depends.

I had thought at first of tabulating τ with arguments R and c for comets, and with log R and c as arguments for minor planets. But as such tables would be too extensive, I have replaced them by others which are shorter. They are given in the present paper.

§ 2. For a cometary orbit, for which R is very small, it is convenient to expand τ in powers of R. Thus, if we put $c^2=\sigma$, we find, for all three kinds of orbits,

$$\tau=\tau_0\mu,$$

$$\tau_0=\left[\frac{3c}{(1+c)^{\frac{3}{2}}-(1-c)^{\frac{3}{2}}}\right]^2=\frac{9}{2}\frac{1+3\sigma+(1-\sigma)^{\frac{3}{2}}}{(3+\sigma)^2},$$

$$\mu=1+R(-\tfrac{1}{4}\sigma-\tfrac{1}{30}\sigma^2-\tfrac{97}{8640}\sigma^3-\tfrac{1375}{252720}\sigma^4-\ldots)+R^2(-\tfrac{1}{4}\sigma+\tfrac{7}{2240}\sigma^2+\tfrac{5}{6720}\sigma^3-\ldots)+R^3(-\tfrac{1}{4}\sigma+\tfrac{7}{2240}\sigma^2+\tfrac{1383}{604800}\sigma^3+\ldots)+R^4(-\tfrac{1}{4}\sigma-\tfrac{1}{30}\sigma^2-\ldots)+R^5(-\tfrac{1}{4}\sigma-\tfrac{19}{1120}\sigma^2-\ldots)+\ldots.$$

Hence

$$\log\tau=\log\tau_0-\frac{R}{1-R}m+\Delta\log\tau,$$

where

$$m=M\frac{45-42\sigma-115\sigma^2-(45+28\sigma-37\sigma^2)\sqrt{1-\sigma}}{45[3+4\sigma+\sigma^2+(3+\sigma)\sqrt{1-\sigma}]},$$

$$\Delta\log\tau=\sigma^2(0{\cdot}013\ 572R^2-0{\cdot}0543R^4-0{\cdot}109R^5)+\sigma^3(0{\cdot}004\ 49R^2+0{\cdot}0031R^3)+\ldots.$$

Table I. gives the values of $\log \tau_0$, m, and $\Delta \log \tau$.

If we consider a planetary orbit (or an orbit of a short-period comet), it is more convenient to use the expansion of τ in powers of σ, for in such case we have usually

$$\sigma < 0{\cdot}05 \quad \text{and} \quad 0{\cdot}1 < R < 0{\cdot}7.$$

This expansion is

$$\tau = 1 + \frac{1 - 4R}{12(1-R)}\sigma + \frac{5 - 28R + 56R^2 - 48R^3}{240(1-R)^3}\sigma^2 + \frac{245 - 1806R + 5496R^2 - 8984R^3 + 8424R^4 - 4320R^5}{30240(1-R)^5}\sigma^3 + \ldots,$$

which gives

$$\log \tau = A\Sigma + B\Sigma^2 + \delta \log \tau, \quad \Sigma = \frac{\sigma}{1-R}.$$

From this formula, by means of Table II., which gives A, B, and $\delta \log \tau$, we may find $\log \tau$.

§ 3. It is of great importance in the explanation that η, the ratio of the triangle, formed by two positions of the body and the sun, to the corresponding sector, may be expressed by the same two quantities R and σ as τ. Indeed, it may be shown that, for an elliptic orbit, for example,

$$\eta = \frac{\sin(\epsilon - \delta) - \sin\epsilon + \sin\delta}{\epsilon - \sin\epsilon - \delta + \sin\delta},$$

where ϵ and δ have the same signification as in (2).

Developing as above, it becomes, for orbits of great eccentricity,

$$\log \eta = \log \eta_0 - \frac{R}{1-R}n + \Delta \log \eta,$$

where

$$\eta_0 = \frac{3}{1 + 2(1-\sigma)^{-\frac{1}{2}}}, \quad n = \frac{2M}{5}\,\frac{1 - (1+2\sigma)(1-\sigma)^{-\frac{1}{2}}}{1 + 2(1-\sigma)^{-\frac{1}{2}}},$$

$$\Delta \log \eta = -\sigma^2(0{\cdot}113\ 40R^2 + 0{\cdot}0241R^3 + 0{\cdot}058R^4 + 0{\cdot}15R^5) - \sigma^3(0{\cdot}030\ 22R^2 + 0{\cdot}0343R^3 + 0{\cdot}066R^4) - \ldots.$$

And, for planetary orbits, we get

$$\eta = 1 - \frac{-1}{3(1-R)}\sigma + \frac{-25 + 82R - 72R^2}{180(1-R)^3}\sigma^2 + \frac{-595 + 3248R - 6964R^2 + 7236R^3 - 3240R^4}{7560(1-R)^5}\sigma^3 + \ldots,$$

and

$$\log \eta = -0{\cdot}144\ 7648\Sigma - C\Sigma^2 + \delta \log \eta, \quad \Sigma = \frac{\sigma}{1-R}.$$

All these quantities $\log \eta_0$, n, $\Delta \log \eta$, C, and $\delta \log \eta$ are given in the Tables I. and II.

§ 4. Let t, t_1, t_2 $(t_1 < t < t_2)$ be the times of observation, α, α_1, α_2, δ, δ_1, δ_2 the corresponding right ascensions and declinations (or longitudes and latitudes), and ρ, ρ_1, ρ_2 the distances of the body from the earth; and let

$$\lambda = \cos\delta\cos\alpha, \quad \mu = \cos\delta\sin\alpha, \quad \nu = \sin\delta, \ \ldots$$

Then we shall have

$$(3) \quad \ldots \quad \begin{cases} x = \rho\lambda - X, \ \ldots, \\ y = \rho\mu - Y, \ \ldots, \\ z = \nu - Z, \ \ldots, \end{cases}$$

where X, Y, Z, . . . are the corresponding sun's co-ordinates.

Putting

$$(4) \quad \ldots \quad \theta = k(t_2 - t_1), \quad \theta_1 = k(t_1 - t), \quad \theta_2 = k(t_2 - t),$$

$$n_1 = \frac{\theta_2\eta_2}{\theta\eta}, \quad n_2 = \frac{\theta_1\eta_1}{\theta\eta},$$

and

$$\begin{aligned} &\lambda_{12} = \mu_1\nu_2 - \mu_2\nu_1, \quad U = X\lambda_{12} - Y\mu_{12} + Z\nu_{12}, \\ &\mu_{12} = \lambda_1\nu_2 - \lambda_2\nu_1, \quad U_1 = X_1\lambda_{12} - Y_1\mu_{12} + Z_1\nu_{12}, \quad \Delta = \lambda\lambda_{12} - \mu\mu_{12} + \nu\nu_{12}, \\ &\nu_{12} = \lambda_1\mu_2 - \lambda_2\mu_1, \quad U_2 = X_2\lambda_{12} - Y_2\mu_{12} + Z_2\nu_{12}, \end{aligned}$$

we get, in the well-known way,

$$(5) \quad \ldots \quad \rho = \frac{1}{\Delta}(U - n_1U_1 + n_2U_2)$$

$$(6) \quad \begin{cases} \rho_1 = \dfrac{1}{n_1}\dfrac{\lambda\mu_2 - \mu\lambda_2}{\nu_{12}}\rho - \dfrac{\mu_2X - \lambda_2Y}{\nu_{12}}\dfrac{1}{n_1} + \dfrac{\mu_2X_1 - \lambda_2Y_1}{\nu_{12}} - \dfrac{\mu_2X_2 - \lambda_2Y_2}{\nu_{12}}\dfrac{n_2}{n_1} \\ \rho_2 = -\dfrac{1}{n_2}\dfrac{\lambda}{\lambda_2}\rho + \dfrac{n_1}{n_2}\dfrac{\lambda_1}{\lambda_2}\rho_1 + \dfrac{X}{\lambda_2}\dfrac{1}{n_2} - \dfrac{X_1}{\lambda_2}\dfrac{n_2}{n_2} + \dfrac{X_2}{\lambda_2}. \end{cases}$$

To obtain the approximate values of the geocentric distances, we may take, as usual,

$$(7) \quad \ldots \quad \begin{cases} \theta\eta = \theta + \gamma r^{-3}, \quad \theta_1\eta_1 = \theta_1 + \gamma_1 r^{-3}, \quad \theta_2\eta_2 = \theta_2 + \gamma_2 r^{-3}, \\ \gamma = -\tfrac{1}{6}\theta^3, \quad \gamma_1 = -\tfrac{1}{6}\theta_1^3, \quad \gamma_2 = -\tfrac{1}{6}\theta_2^3. \end{cases}$$

Substituting in (4) and (5) these values, we get

$$(8) \quad \ldots \quad \rho = P + (Q + q\rho)r^{-3}$$

where

$$P = \frac{D'}{\theta\Delta}, \quad Q = \frac{D''}{\theta\Delta}, \quad q = \tfrac{1}{6}\theta^2,$$

$$D' = \theta U - \theta_2U_1 + \theta_1U_2, \quad D'' = \gamma U - \gamma_2U_1 + \gamma_1U_2.$$

When ρ and r are found from (8) and

$$(9) \quad \ldots \quad r^2 = \rho^2 - 2\rho(\lambda X + \mu Y + \nu Z) + X^2 + Y^2 + Z^2,$$

and ρ_1, ρ_2 from (7), (4), (6), then we may obtain, by means of equations (3), the heliocentric co-ordinates of the body for the times of observation; and from

$$r^2 = x^2 + y^2 + z^2, \quad r_1^2 = x_1^2 + y_1^2 + z_1^2, \quad r_2^2 = x_2^2 + y_2^2 + z_2^2,$$
$$s_1^2 = (x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2, \quad s_2^2 = (x_2 - x)^2 + (y_2 - y)^2 + (z_2 - z)^2,$$
$$s^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2,$$

the corresponding radii-vectores and the chords.

Taking now two of the positions of the body, first and second, for instance, we may compute a from

$$\frac{1}{4a} = \frac{\tau}{r_1 + r} - \frac{s_1^2}{4\theta_1^2}.$$

Then, deriving η, η_1, η_2 from tables with arguments

$$R = \frac{r_1 + r_2}{4a}, \quad R_1 = \frac{r_1 + r}{4a}, \quad R_2 = \frac{r_2 + r}{4a},$$
$$\sigma = \left(\frac{s}{r_1 + r_2}\right)^2, \quad \sigma_1 = \left(\frac{s_1}{r_1 + r}\right)^2, \quad \sigma_2 = \left(\frac{s_2}{r_2 + r}\right)^2,$$

and computing from (4), (5), (6) the new values of ρ, ρ_1, ρ_2, we shall have more approximate values of heliocentric co-ordinates x_1, y_1, z_1, x_2, y_2, z_2, from which, if the third approximation is not necessary, we may obtain a, by means of Lambert's theorem, and other elements.

§ 5. To illustrate the numerical application of the formulæ thus derived, let us take the following observations of *Eleonora* (354) made at Marseilles by Esmiol (*Bull. Astr.*, t. 29):—

	Gr. M.T.	α 1912·0.	δ 1912·0.
$t_1 =$ 1912	March 25·329 484	156° 3′ 27″·8	+21° 58′ 14″·9
$t =$	April 23·319 172	155 48 23 ·2	+23 20 19 ·2
$t_2 =$	May 22·401 857	160 31 46 ·1	+21 50 58 ·4

These places are freed from star-aberration. The corresponding direction cosines and topocentric co-ordinates of the sun are

$\lambda_1 = -0{\cdot}847\,578$,	$\lambda = -0{\cdot}837\,533$,	$\lambda_2 = -0{\cdot}875\,086$.
$\mu_1 = +0{\cdot}376\,343$,	$\mu = +0{\cdot}376\,289$,	$\mu_2 = +0{\cdot}309\,378$.
$\nu_1 = +0{\cdot}374\,134$,	$\nu = +0{\cdot}396\,165$,	$\nu_2 = +0{\cdot}372\,171$.
$X_1 = +0{\cdot}994\,305$,	$X = +0{\cdot}841\,103$,	$X_2 = +0{\cdot}484\,916$.
$Y_1 = +0{\cdot}076\,785$,	$Y = +0{\cdot}506\,281$,	$Y_2 = +0{\cdot}815\,616$.
$Z_1 = +0{\cdot}033\,293$,	$Z = +0{\cdot}219\,606$,	$Z_2 = +0{\cdot}353\,791$.

In the present case we get

$$\Delta = +0{\cdot}001\,7234; \quad U = +0{\cdot}029\,1365; \quad U_1 = +0{\cdot}025\,4929; \quad U_2 = +0{\cdot}025\,7828.$$

$$\rho_1 = \frac{1{\cdot}04\,560}{n_1}\rho - \frac{10{\cdot}47\,911}{n_1} + 5{\cdot}58\,496 - 12{\cdot}87\,066\frac{n_2}{n_1}.$$

$$\rho_2 = -\frac{0{\cdot}957\,087}{n_2}\rho + 0{\cdot}968\,566\frac{n_1}{n_2}\rho - \frac{0{\cdot}961\,166}{n_2} + 1{\cdot}136\,237\frac{n_1}{n_2} - 0{\cdot}554\,135.$$

The equations (8) and (9) give $\rho = 1{\cdot}9116$, $r^{-3} = 0{\cdot}063\ 27$. Therefore we have $n_1 = +0{\cdot}504\ 790$, $n_2 = -0{\cdot}503\ 183$, and $\rho_1 = 1{\cdot}614\ 91$, $\rho_2 = 2{\cdot}283\ 02$ With these values of geocentric distances we obtain

$t_1' = 25{\cdot}320\ 166$, $r_1 \ldots 0{\cdot}39\ 591$, $s_1^2 \ldots 9{\cdot}03\ 521$, $\sigma_1 \ldots 7{\cdot}63\ 761$.
$t' = 54{\cdot}308\ 142$, $r \ldots 0{\cdot}39\ 962$, $s^2 \ldots 9{\cdot}63\ 338$, $\sigma \ldots 8{\cdot}23\ 154$.
$t_2' = 83{\cdot}388\ 684$, $r_2 \ldots 0{\cdot}40\ 383$, $s_2^2 \ldots 9{\cdot}03\ 058$, $\sigma_2 \ldots 7{\cdot}62\ 506$.

The value of a must now be found from the equation

$$\frac{1}{4a} = \frac{\tau}{r_1 + r} - [9{\cdot}03\ 755].$$

To obtain an approximate value of R_1, we put $\tau = 1$, and consequently

$$R_1 = \frac{r_1 + r}{4a} = 1 - \frac{s_1^2}{4\theta_1^2}(r_1 + r),$$

which gives

$$R_1 = 0{\cdot}45\ 506, \quad \Sigma_1 = 0{\cdot}00\ 796.$$

Thus, from Table II., $A = -0{\cdot}0297$, $B = 0{\cdot}00$, $\delta \log \tau = 0$, and therefore $\tau \ldots 9{\cdot}99\ 976$. Using this value, we obtain $\frac{1}{4a} \ldots 8{\cdot}95\ 875$, $R_1 = 0{\cdot}45\ 452$. A repetition of the calculation does not change these values.

Thus we get

$\Sigma_1 = 0{\cdot}007\ 958$, $C_1 = 0{\cdot}036$, $\delta \log \eta = 0$, $\eta_1 \ldots 9{\cdot}998\ 846$.
$\Sigma_2 = 0{\cdot}007\ 791$, $C_2 = 0{\cdot}035$, $= 0$, $\eta_2 \ldots 9{\cdot}998\ 870$.
$\Sigma = 0{\cdot}031\ 371$, $C = 0{\cdot}036$, $= -0{\cdot}5$, $\eta \ldots 9{\cdot}995\ 424$.

Therefore $n_1 = +0{\cdot}50\ 4787$, $n_2 = 0{\cdot}50\ 3153$; and, from (5) and (6), $\rho = 1{\cdot}9121$, $\rho_1 = 1{\cdot}61\ 513$, $\rho_2 = 2{\cdot}28\ 392$.

Finding now the elements from the first and third places, we obtain

1912 April 23·0 Gr.M.T. $M = 34^\circ\ 59'\ 56''{\cdot}3$.

$$\left.\begin{array}{llll} \phi & = 6^\circ\ 25'\ 14''{\cdot}3 & i = 14^\circ\ 30'\ 14''{\cdot}4 & \\ a \ldots & 0{\cdot}43\ 9117 & \Omega = 53\ \ 0\ 44\ {\cdot}5 & \\ n & = 778''{\cdot}630 & \omega = 77\ 49\ 37\ {\cdot}2 & \end{array}\right\} 1912{\cdot}0.$$

These elements represent the observations on which they are based as follows:—

	Comp. − Obs.		
A.R.	$-0''{\cdot}1$,	$+0''{\cdot}1$,	$+0''{\cdot}2$,
Decl.	$+0''{\cdot}5$,	$+0''{\cdot}1$,	$-0''{\cdot}2$.

It appears, from this example, that in the computation of a planetary orbit in usual conditions no tables are necessary, and we may be content with the simple approximate formulæ

$$\log\tau = -0{\cdot}036\ 1912(4R-1)\Sigma$$
$$\log\eta = -0{\cdot}144\ 7648\Sigma - 0{\cdot}035\Sigma^2.$$

It may be also advantageous to use the following approximate formulæ

$$\tau_0 = (1 - \tfrac{5}{12}\sigma)^{-\frac{1}{3}}, \quad \eta_0 = (1 - \tfrac{7}{6}\sigma)^{\frac{2}{7}}.$$

§ 6. The indicated form of Lambert's theorem may be useful, not only in computing a new orbit, but also in correcting an orbit approximately known, by varying two geocentric distances.

Let us designate $\lambda', \mu', \nu', X', \ldots, x', y', \ldots, \lambda'', \mu'', \ldots$ the quantities relative to the places, whose geocentric distances must be varied. Let $\lambda_i, \mu_i, \ldots$ be the numbers relative to any other observation. We find easily

$$\rho_i = \frac{M_i\rho'\rho'' + N_i\rho' + P_i\rho'' + Q_i}{A_i\rho'\rho'' + B_i\rho' + C_i\rho'' + D_i},$$

where

$$A_i = \lambda\lambda_i - \mu\mu_i + \nu\nu_i, \qquad M_i = \lambda X_i - \mu Y_i + \nu Z_i,$$
$$B_i = l'\lambda_i - m'\mu_i + n'\nu_i, \qquad N_i = l'X_i - m'Y_i + n'Z_i,$$
$$C_i = l''\lambda_i - m''\mu_i + n''\nu_i, \qquad P_i = l''X_i - m''Y_i + n''Z_i,$$
$$D_i = X\lambda_i - Y\mu_i + Z\nu_i, \qquad Q_i = XX_i - YY_i + ZZ_i,$$

and

$$\lambda = \mu'\nu'' - \mu''\nu', \quad l' = \nu'Y'' - \mu'Z'', \quad l'' = \mu''Z' - \nu''Y', \quad X = Y'Z'' - Y''Z'.$$
$$\mu = \lambda'\nu'' - \lambda''\nu', \quad m' = \nu'X'' - \lambda'Z'', \quad m'' = \lambda''Z' - \nu''X', \quad Y = X'Z'' - X''Z'.$$
$$\nu = \lambda'\mu'' - \lambda''\mu', \quad n' = \mu'X'' - \lambda'Y'', \quad n'' = \lambda''Y' - \mu''X', \quad Z = X'Y'' - X''Y'.$$

Therefore all heliocentric co-ordinates, and consequently all radii-vectores and chords, joining every two positions of the body may be regarded as being expressed by means of ρ' and ρ''.

Using now Lambert's theorem, we shall have for each pair of positions an equation containing ρ', ρ'', and a. It is clear that we may consider these equations as being of the form

$$\frac{1}{4a} = \phi_i(\rho', \rho''), \quad \frac{1}{4a} = \psi_i(\rho', \rho''), \quad \frac{1}{4a} = \chi_i(\rho', \rho'')$$

(we have three equations at least for every intermediate place). Eliminating $\frac{1}{4a}$, there remain two equations, at least, for every intermediate place, containing ρ' and ρ''. To resolve these equations we may proceed in the usual way, making three hypotheses upon ρ' and ρ''.

I hope to publish in the future a detailed example of correcting an orbit by this method in the case of numerous observations, and to add to this example the more complete table of $\delta\log\tau$.

Novotsherkassk:
1922 *April* 17.

Table I.

For finding $\log\tau$ *and* $\log\eta$ *in Orbits of Great Eccentricity.*

σ.	$\log\tau_0$.	m.	$\log\eta_0$.	n.	σ.	$\log\tau_0$.	m.	$\log\eta_0$.	n.
0·00	0·000 0000	0·000 000	0·000 0000	0·000 000	0·25	0·009 5700	0·028 135	9·957 3719	0·038 427
·01	·000 3627	·001 087	9·998 5439	·001 451	·26	·009 9771	·029 306	·955 3358	·040 071
·02	·000 7269	·002 177	·997 0704	·002 908	·27	·010 3862	·030 481	·953 2678	·041 725
·03	·001 0926	·003 270	·995 5794	·004 372	·28	·010 7976	·031 659	·951 1670	·043 389
·04	·001 4599	·004 366	·994 0704	·005 842	·29	·011 2110	·032 842	·949 0324	·045 063
0·05	0·001 8288	0·005 466	9·992 5429	0·007 319	0·30	0·011 6267	0·034 029	9·946 8630	0·046 747
·06	·002 1992	·006 568	·990 9966	·008 802	·31	·012 0446	·035 220	·944 6577	·048 442
·07	·002 5713	·007 673	·989 4310	·010 293	·32	·012 4648	·036 415	·942 4155	·050 147
·08	·002 9451	·008 781	·987 8458	·011 790	·33	·012 8873	·037 615	·940 1352	·051 863
·09	·003 3204	·009 893	·986 2404	·013 294	·34	·013 3121	·038 818	·937 8155	·053 590
0·10	0·003 6975	0·011 007	9·984 6144	0·014 806	0·35	0·013 7393	0·040 026	9·935 4553	0·055 329
·11	·004 0763	·012 125	·982 9673	·016 325	·36	·014 1689	·041 239	·933 0532	·057 079
·12	·004 4568	·013 246	·981 2985	·017 851	·37	·014 6010	·042 456	·930 6078	·058 841
·13	·004 8390	·014 371	·979 6076	·019 385	·38	·015 0356	·043 677	·928 1177	·060 615
·14	·005 2230	·015 498	·977 8941	·020 927	·39	·015 4727	·044 903	·925 5813	·062 401
0·15	0·005 6087	0·016 629	9·976 1573	0·022 476	0·40	0·015 9124	0·046 134	9·922 9970	0·064 201
·16	·005 9963	·017 764	·974 3967	·024 033	·41	·016 3548	·047 370	·920 3632	·066 013
·17	·006 3857	·018 902	·972 6116	·025 598	·42	·016 7998	·048 610	·917 6780	·067 838
·18	·006 7770	·020 043	·970 8015	·027 172	·43	·017 2476	·049 856	·914 9397	·069 678
·19	·007 1701	·021 188	·968 9658	·028 753	·44	·017 6981	·051 106	·912 1462	·071 531
0·20	0·007 5652	0·022 337	9·967 1036	0·030 343	0·45	0·018 1514	0·052 362	9·909 2954	0·073 399
·21	·007 9622	·023 489	·965 2144	·031 942	·46	·018 6076	·053 622	·906 3851	·075 281
·22	·008 3611	·024 645	·963 2974	·033 550	·47	·019 0668	·054 888	·903 4131	·077 179
·23	·008 7620	·025 804	·961 3518	·035 166	·48	·019 5289	·056 159	·900 3768	·079 092
·24	·009 1650	·026 968	·959 3769	·036 792	·49	·019 9941	·057 436	·897 2736	·081 021
0·25	0·009 5700	0·028 135	9·957 3719	0·038 427	0·50	0·020 4624	0·058 718	9·894 1009	0·082 966

$$\log\tau=\log\tau_0-\frac{\mathrm{R}}{1-\mathrm{R}}m+\Delta\log\tau\,;\quad \log\eta=\log\eta_0-\frac{\mathrm{R}}{1-\mathrm{R}}n+\Delta\log\eta\,;\quad \mathrm{R}=\frac{r+r'}{4a}\,,\ \sigma=\left(\frac{s}{r+r'}\right)^2.$$

TABLE I.—*continued.*

Values of $10^7 \Delta \log \tau$.

R/σ	±0·01.	±0·02.	±0·03.	±0·04.	±0·05.	±0·06.	±0·07.	±0·08.	0·09.	0·10.	−0·09.	−0·10.
0·01	+0·0	+0·0	+0·0	+0·0	+ 0·0	+ 0·0	+ 0·1	+ 0·1	+ 0·1	+ 0·1	+ 0·1	+ 0·1
·02	0·0	0·0	0·0	0·1	0·1	0·2	0·3	0·3	0·4	0·5	0·4	0·5
·03	0·0	0·0	0·1	0·2	0·3	0·4	0·6	0·8	1·0	1·2	1·0	1·2
·04	0·0	0·1	0·2	0·3	0·5	0·8	1·1	1·4	1·7	2·1	1·7	2·1
0·05	0·0	0·1	0·3	0·5	0·8	1·2	1·7	2·1	2·7	3·3	2·7	3·3
·06	0·1	0·2	0·4	0·8	1·2	1·8	2·4	3·1	3·9	4·7	3·9	4·8
·07	0·1	0·3	0·6	1·1	1·7	2·4	3·3	4·2	5·3	6·5	5·3	6·6
·08	0·1	0·4	0·8	1·4	2·2	3·2	4·3	5·6	6·9	8·5	7·0	8·6
·09	0·1	0·5	1·0	1·8	2·8	4·0	5·5	7·0	8·8	10·8	8·9	10·9
0·10	0·1	0·6	1·3	2·2	3·5	5·0	6·7	8·7	11·0	13·4	11·0	13·6
·11	0·2	0·7	1·5	2·7	4·2	6·0	8·2	10·6	13·3	16·3	13·4	16·4
·12	0·2	0·8	1·8	3·2	5·0	7·2	9·8	12·6	15·9	19·4	16·0	19·6
·13	0·3	0·9	2·1	3·8	5·9	8·5	11·5	14·9	18·7	22·9	18·8	23·1
·14	0·3	1·1	2·5	4·4	6·9	9·9	13·4	17·3	21·8	26·6	21·9	26·9
0·15	0·3	1·3	2·9	5·1	7·9	11·4	15·4	20·0	25·2	30·7	25·2	31·0
·16	0·4	1·5	3·3	5·8	9·0	13·0	17·6	22·8	28·6	35·0	28·8	35·4
·17	0·4	1·7	3·7	6·6	10·2	14·7	19·9	25·9	32·4	39·7	32·6	40·0
·18	0·5	1·9	4·2	7·4	11·5	16·5	22·4	29·1	36·5	44·7	36·7	45·0
·19	0·5	2·1	4·7	8·3	12·9	18·5	25·1	32·5	40·8	49·9	41·0	50·3
0·20	+0·6	+2·3	+5·2	+9·2	+14·3	+20·6	+27·8	+36·1	+45·4	+55·5	+45·5	+55·9

TABLE I.—*continued.*

Values of $10^7 \Delta \log \eta$.

R/σ	−0·05.	−0·04.	−0·03.	−0·02.	−0·01.	+0·01.	+0·02.	+0·03.	+0·04.	+0·05.	+0·06.	+0·07.	+0·08.	+0·09.	+0·10.
0·01	− 0·3	− 0·2	− 0·1	− 0·0	−0·0	−0·0	− 0·0	− 0·1	− 0·2	− 0·3	− 0·4	− 0·6	− 0·7	− 0·9	− 1·2
·02	1·1	0·7	0·4	0·2	0·0	0·0	0·2	0·4	0·7	1·2	1·7	2·3	3·0	3·8	4·7
·03	2·5	1·6	0·9	0·4	0·1	0·1	0·4	0·9	1·7	2·6	3·8	5·1	6·7	8·5	10·6
·04	4·5	2·9	1·6	0·7	0·2	0·2	0·7	1·7	3·0	4·6	6·7	9·2	12·0	15·3	18·9
0·05	− 7·1	− 4·5	− 2·6	− 1·2	−0·3	−0·3	− 1·2	− 2·6	− 4·6	− 7·3	− 10·5	− 14·3	− 18·8	− 23·8	− 29·6
·06	10·3	6·6	3·7	1·7	0·4	0·4	1·7	3·7	6·7	10·5	15·2	20·7	27·1	34·4	42·7
·07	14·0	9·0	5·1	2·3	0·6	0·6	2·3	5·1	9·1	14·3	18·7	28·2	37·0	47·0	58·3
·08	18·3	11·8	6·6	3·0	0·7	0·7	3·0	6·7	12·0	18·8	27·1	37·0	48·5	61·6	76·3
·09	23·3	14·9	8·4	3·7	0·9	0·9	3·8	8·5	15·2	23·8	34·4	47·0	61·5	78·2	96·9
0·10	−28·8	−18·5	−10·4	− 4·6	−1·2	−1·2	− 4·7	−10·6	−18·8	−29·5	− 42·6	− 58·1	− 76·2	− 96·8	−120·0
·11	34·9	22·4	12·6	5·6	1·4	1·4	5·7	12·8	22·8	35·8	51·7	70·5	92·4	117·4	145·6
·12	41·7	26·7	15·1	6·7	1·7	1·7	6·8	15·3	27·2	42·7	61·8	84·2	110·3	140·1	173·8
·13	49·0	31·4	17·7	7·9	2·0	2·0	8·0	18·0	32·1	50·2	72·6	99·0	129·9	164·9	204·5
·14	57·0	36·6	20·6	9·2	2·3	2·3	9·3	20·9	37·3	58·4	84·4	115·2	151·0	191·8	237·8
0·15	−67·6	−44·1	−23·7	−11·6	−2·6	−2·7	−11·7	−24·1	−42·9	−67·3	−107·2	−132·6	−174·8	−221·8	−273·8

TABLE II.

For finding $\log\tau$ *and* $\log\eta$ *in Elliptic Orbits.*

R.	A.	B.	C.	R.	A.	B.	C.
0·10	+0·021 7147	+0·004 910	+0·071 096	0·40	−0·021 7147	−0·001 484	+0·039 087
·11	·020 2671	·004 679	·069 810	·41	·023 1624	·001 767	·038 372
·12	·018 8194	·004 452	·068 535	·42	·024 6100	·002 063	·037 692
·13	·017 3718	·004 230	·067 271	·43	·026 0577	·002 375	·037 050
·14	·015 9241	·004 013	·066 017	·44	·027 5053	·002 704	·036 446
0·15	+0·014 4765	+0·003 799	+0·064 775	0·45	−0·028 9530	−0·003 051	+0·035 884
·16	·013 0288	·003 590	·063 545	·46	·030 4006	·003 418	·035 366
·17	·011 5812	·003 383	·062 327	·47	·031 8483	·003 807	·034 893
·18	·010 1335	·003 180	·061 121	·48	·033 2959	·004 220	·034 469
·19	·008 6859	·002 979	·059 929	·49	·034 7436	·004 660	·034 096
0·20	+0·007 2382	+0·002 781	+0·058 750	0·50	−0·036 1912	−0·005 127	+0·033 778
·21	·005 7906	·002 584	·057 586	·51	·037 6389	·005 626	·033 518
·22	·004 3429	·002 389	·056 436	·52	·039 0865	·006 158	·033 320
·23	·002 8953	·002 196	·055 301	·53	·040 5342	·006 726	·033 187
·24	+0·001 4476	·002 003	·054 183	·54	·041 9818	·007 335	·033 124
0·25	0·000 0000	+0·001 810	+0·053 080	0·55	−0·043 4294	−0·007 986	+0·033 135
·26	−0·001 4476	·001 616	·051 995	·56	·044 8771	·008 685	·033 226
·27	·002 8953	·001 422	·050 928	·57	·046 3247	·009 435	·033 401
·28	·004 3429	·001 227	·049 880	·58	·047 7724	·010 240	·033 668
·29	·005 7906	·001 029	·048 850	·59	·049 2200	·011 107	·034 033
0·30	−0·007 2382	+0·000 829	+0·047 841	0·60	−0·050 6677	−0·012 040	+0·034 502
·31	·008 6859	·000 626	·046 853	·61	·052 1153	·013 045	·035 093
·32	·010 1335	·000 418	·045 888	·62	·053 5630	·014 129	·035 790
·33	·011 5812	+0·000 206	·044 945	·63	·055 0106	·015 299	·036 627
·34	·013 0288	−0·000 012	·044 026	·64	·056 4583	·016 564	·037 607
0·35	−0·014 4765	−0·000 236	+0·043 132	0·65	−0·057 9059	−0·017 934	+0·038 742
·36	·015 9241	·000 467	·042 265	·66	·059 3536	·019 417	·040 046
·37	·017 3718	·000 706	·041 426	·67	·060 8012	·021 026	·041 534
·38	·018 8194	·000 955	·040 625	·68	·062 2489	·022 774	·043 224
·39	·020 2671	·001 214	·039 835	·69	·063 6965	·024 674	·045 135
0·40	−0·021 7147	−0·001 484	+0·039 087	0·70	−0·065 1442	−0·026 745	+0·047 290

$$\log\tau = A\Sigma + B\Sigma^2 + \delta\log\tau\,; \quad \log\eta = -0{\cdot}144\ 7648\ \Sigma - C\Sigma^2 + \delta\log\eta\,; \quad \Sigma = \frac{\sigma}{1-R}.$$

σ=	0·01.	0·02.	0·03.	0·04.	0·05.	0·01.	0·02.	0·03.	0·04.
R.	$10^7\delta\log\tau$.					$10^7\delta\log\eta$.			
0·10	+0·0	+0·2	+0·6	+1·5	+2·9	−1	−5	−17	−39
·20	0·0	0·1	0·5	1·2	2·3	1	5	17	40
·30	+0·0	+0·1	+0·3	+0·7	+1·4	1	5	18	42
·40	−0·0	−0·0	−0·2	−0·4	−0·7	1	6	19	45
·45	0·0	0·3	0·9	2·1	4·1	1	6	20	48
·50	0·1	0·8	2·8	6·5	12·8	1	7	23	56
·55	0·3	2·3	7·8	18·6	36·2	1	9	31	74
0·60	−0·8	−6·6	−22·4	−53·1	−103·6	−2	−16	−53	−125

Observations made during the Partial Eclipse of the Sun on 1922 *March* 28 *at the University Observatory, Rostov-on-Don.* By Professor S. D. Tscherny.

(*Communicated by the Secretaries.*)

Phenomenon.	Telescope.	Rostov-on-Don. Mean Time. h m s	Observer.	Remarks.
First contact	V	4 23 58·3	S.T.	A little late.
,, ,,	T	4 23 55·1	A.B.	,, ,, ,,
,, ,,	B	4 23 56·1	W.D.	,, ,, ,,
Last contact	V	6 19 50·3	S. T.	The last contact was observed on the horizon. The limbs of the Sun and the Moon were boiling.

Instruments used:—

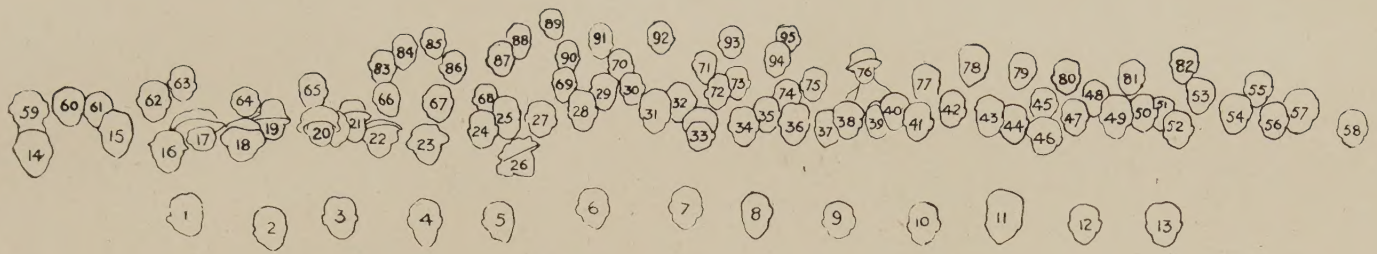
V. Vion refractor, 71 mm., power 71 (first contact) and 24 (last contact).
T. Telescope 52 mm., power 26.
B. Butenschön altazimuth 30 mm., power 31.

Observers.	Position of Observatory.
S.T. Professor S. Tscherny.	
A.B. Student A. Batirew.	Latitude, 47° 13′ 9″·4 N.
W.D. ,, W. Davidson.	Longitude, $2^h\ 38^m\ 49^s{\cdot}48$ E.

University Observatory,
Rostov-on-Don:
1922 *April* 1.

ROYAL ASTRONOMICAL SOCIETY CENTENARY CELEBRATION, 1922 MAY 30.

Photograph by F. A. Swaine,
146 New Bond Street, W.



1. Seares.
2. Knobel.
3. Aitken.
4. Hertzsprung.
5. Shapley.
6. Phillips (*Secretary*).
7. Eddington (*President*).
8. Crommelin (*Secretary*).
9. St. John.
10. S. S. Hough.
11. Strömgren.
12. Fowler.
13. Hollis.
14. Olsen.
15. Dreyer.
16. Miss Cook.
17. Mrs. Dodwell.
18. Mrs. Freeman.
19. Lady Dyson.
20. Countess de Rivas.
21. Miss Cave-Browne-Cave.
22.
23. Mrs. Shapley.
24. Dodwell.
25. Lindemann.
26. Miss Herschel.
27. Miller.
28. Grove-Hills (*Treas.*).
29. Lee.
30. Lockyer.
31. Turner (*For. Sec.*).
32. Newall.
33. Maharajah of Jalawar.
34. Gwilliam.
35. Davies.
36. Dyson (*Astron. Royal*).
37.
38. Rendell.
39. Cottingham.
40. Hony.
41. Byfield.
42. Maxwell.
43. Waran.
44. Heath.
45.
46. Klotz.
47. Bell.
48. Baxandall.
49. Thorrowgood.
50. Burnet.
51. Milne.
52. Dingle.
53. Greaves.
54. Purey-Cust.
55 Marshall.
56. Smart.
57 Robbins.
58. Bartrum.
59. Inwards.
60. Hepburn.
61.
62. Thomson.
63. Arnold.
64. Phayre.
65. Adames.
66. Bellamy.
67. Herschel.
68. Knox-Shaw.
69. Jackson.
70. Bower.
71. Allen.
72. Waterfield.
73. Witchell.
74. Levin.
75.
76. Mrs. Merton.
77. Merton.
78. Ainslie.
79. Longbottom.
80. Steavenson.
81. Comrie.
82. Lenox-Conyngham.
83. Hudson.
84. Leeson.
85. Chamberlin.
86. Parr.
87. Markwick.
88. Goldsbrough.
89. Plummer.
90. Newbegin.
91. Fisher.
92. Stoney-Smith.
93. Butler.
94. Bryant.
95. Hall.

Key to Plate 11.

The Secretaries would be glad to receive the names that are missing in the above key.